UNIVERSITÀ DEGLI STUDI FIRENZE

DISEI

DIPARTIMENTO DI SCIENZE PER L'ECONOMIA E L'IMPRESA

Working Papers - Economics

On Bundling and Entry Deterrence

Andrea Greppi, Domenico Menicucci

Working Paper N. 26/2018

(Second Version, January 2020)

DISEI, Università degli Studi di Firenze

Via delle Pandette 9, 50127 Firenze (Italia) www.disei.unifi.it

1 Introduction

This paper investigates how the possibility to bundle helps a multiproduct firm to deter entry, depending on the sort of competition faced by the bundling firm. Precisely, we consider a firm A offering two products, $A1$ and $A2$, with higher quality than its rivals' products – for this reason we say that A is *dominant* over its rivals – and use a parameter $\alpha \geq 0$ to represent the quality difference. In a bidimensional Hotelling environment, we compare two games: one in which A faces the threat of entry of a single generalist rival B, offering products $B1$ and $B2$, and another game in which A faces the threat of entry of two single-product rivals, a firm $B1$ offering only product $B1$ and a firm $B2$ offering only product $B2$. After A's rival(s) entry decision(s), A chooses whether to bundle or not its products, and then price competition takes place. We denote with Γ^g the game in which A faces a generalist rival, with Γ^s the game in which A faces specialist rivals. We compare Γ^g and Γ^s to see how A's possibility of bundling affects the entry of A's rival(s), firms' profits, consumers' surplus and social welfare. In particular, we identify the game in which entry is deterred more effectively, and how this depends on α.

Whinston (1990) shows that in some cases a multiproduct firm can use bundling to build an entry barrier. In the baseline model of Whinston (1990), the firm that can bundle faces the threat of entry in a single market and bundling reduces the entrant's profit, possibly below its entry cost. However, if entry occurs then bundling reduces also the profit of the multiproduct firm, thus the latter would prefer to unbundle and some commitment ability is necessary for bundling to deter entry.[1] Hurkens, Jeon and Menicucci (2018) (HJM henceforth) consider a setting in which a multiproduct firm faces competition in all markets in which it is active, and in fact the model we employ in this paper is the one examined in HMJ. They prove – under suitable conditions – that when firm A faces firm B, the effect of bundling on firms' profits depends on α as follows: There exist $\hat{\alpha}, \bar{\alpha}$ such that $0 < \hat{\alpha} < \bar{\alpha}$ and (i) given α smaller than $\hat{\alpha}$, for both A and B the profit decreases under bundling; (ii) if α is larger than $\bar{\alpha}$, then for both firms the profit increases under bundling; (iii) for α between $\hat{\alpha}$ and $\bar{\alpha}$, bundling increases A's profit but reduces B's profit.[2] In the latter case, if firm B has to pay an entry cost before it can compete, then B expects that entry will be followed by A's bundling, and this may reduce B's gross profit below B's entry cost, inducing B to stay out. Unlike in the model of Whinston (1990) mentioned above, here the bundling firm is willing to bundle after it learnt that its opponent has entered, hence no commitment ability is needed. HJM also consider the

[1] Nalebuff (2004) and Peitz (2008) examine settings in which the bundling firm is willing to bundle even after entry has occurred.

[2] Hahn and Kim (2012) obtain similar results in a setting in which firms have different marginal costs, and under more restrictive assumptions than HJM on the distribution of consumers' preferences. Matutes and Regibeau (1988) prove that bundling reduces the firms' profit when firms are symmetric.

case in which A faces specialist firms, and show that the specialists' lack of coordination makes competition under bundling more profitable for A than competition under bundling between A and B. Hence, there exists $\tilde{\alpha}$ such that bundling increases A's profit if $\alpha > \tilde{\alpha}$, and $\tilde{\alpha}$ is smaller than $\hat{\alpha}$.

In this paper we extend the analysis of HJM by examining the specialists' entry choices in Γ^s. These involve a strategic interaction between specialists – unlike in the case of a single rival for A – because, for instance, a specialist's entry may favor (or may discourage) the entry of the other specialist. Our results allow to compare Γ^g with Γ^s to see whether a generalist firm or two specialist firms is/are in a better position to withstand bundling by a dominant firm.

A crucial role in the comparison is played by $\tilde{\alpha}$ and $\hat{\alpha}$ introduced above. When $\alpha < \tilde{\alpha}$, bundling is never profitable for A conditional on the entry of A's rival(s) (we consider this qualification as implied in the rest of this introduction) and then there is no link between the markets for the two products. In this case it is irrelevant whether A faces firm B or firms $B1, B2$, hence Γ^g and Γ^s lead to the same outcome.[3]

When α is between $\tilde{\alpha}$ and $\hat{\alpha}$, firm A bundles in Γ^s but not in Γ^g. It turns out that bundling reduces the profits of $B1, B2$ given $\alpha \in (\tilde{\alpha}, \hat{\alpha})$, hence it contributes to foreclose A's rivals in Γ^s and makes more likely that A is monopolist in one or both markets.[4] Also in Γ^g bundling would help A to deter entry by reducing B's profit, but it is crucial that A's decision about bundling comes after B's decision about entry, and once B has entered, firm A does not want to bundle. In fact, bundling in Γ^g would be even more effective than in Γ^s in discouraging entry as it reduces B's profit more than it reduces the sum of the profits of $B1, B2$ in Γ^s. This occurs because under bundling, in Γ^s firms $B1, B2$ are less aggressive than firm B in Γ^g since each specialist fails to internalize the negative externality of an own product price increase on the other specialist. This benefits firm A as remarked by HJM, but given that prices are strategic complements, firm A increases the price of its bundle as a consequence. In our setting this more than compensates for the specialists' lack of coordination and makes their total profit higher than the generalist' profit. As a consequence, competition under bundling is more fierce in Γ^g than in Γ^s. Notwithstanding the reduction in profit firm B would suffer from bundling, in practice this is not a concern for B when $\alpha \in (\tilde{\alpha}, \hat{\alpha})$ since firm A wants to avoid the tough competition under bundling in Γ^g.

When α is greater than $\hat{\alpha}$, A bundles both in Γ^g and in Γ^s and then firm B cannot escape competition under bundling. If the entry cost is so small that entry occurs in Γ^g and in Γ^s, then each firm prefers Γ^s because of the more favorable competition environment. The firms' higher profits make consumers' surplus lower in Γ^s, but social welfare is greater in Γ^s because

[3] A same outcome is obtained also if the entry cost is very high, because then no rival of A ever enters.

[4] For some parameter values, each specialist randomizes over the entry decision. Hence it may occur that (for instance) only firm $B1$ enters.

A has higher market share than in Γ^g (due to less aggressive pricing by $B1, B2$), hence more consumers buy the higher quality products of A.

For suitably larger (not too much) entry cost, in Γ^g entry is unprofitable for B, but $B1, B2$ still enter in Γ^s because they earn a higher joint profit than B from competing against A under bundling. In this case, bundling is a better entry deterrent in Γ^g than in Γ^s because it does not harm firm A in case of entry, and it is more effective in reducing the profit of firm B in Γ^g than the profits of firms $B1, B2$ in Γ^s.[5] Thus, bundling is more effective in foreclosing a generalist firm than specialist firms. From the point of view of social welfare, Γ^g is superior to Γ^s because no entry cost is incurred, and (almost) all consumers consume the higher quality products of firm A. However, consumers' surplus is very low in Γ^g because firm A does not face competition and can set high prices.

Our results suggest that a competition authority interested in protecting consumers' surplus may want to favor a merger between specialist firms facing a dominant firm when the dominance level α is not too large, but should favor splitting an integrated firm when α is somewhat larger, as an integrated firm is relatively more vulnerable to bundling.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 is about game Γ^g, Section 4 is about game Γ^s,[6] and Section 5 compares the two games. Section 6 concludes.

2 The setting

We consider two competition settings. In one of them, a multiproduct firm A, which offers products $A1$ and $A2$, is already active and faces the threat of entry of two single-product firms $B1$ and $B2$ such that firm Bj offers only product Bj, for $j = 1, 2$. Each firm Bj incurs a cost $k > 0$ if it decides to enter. In the other competition setting, firms $B1$ and $B2$ are merged into a single entity denoted B, which offers products $B1$ and $B2$. Firm A faces the threat of entry of firm B, which incurs a cost $2k$ if it decides to enter.

After his rival(s) entry decisions(s), firm A chooses between offering $A1, A2$ separately or as a bundle – in the latter case, each consumer either buys products $A1$ and $A2$ or buys no product offered by A – that is, A chooses between independent pricing and pure bundling. At the successive (and final) stage, firm A competes in prices with its active rival(s), or acts as a monopolist if no rival entered. We use Γ^s to denote the game in which A competes with

[5] In fact, this claim applies for α not too larger than $\hat{\alpha}$, because for a large α bundling actually increases the profit of B in Γ^g with respect to B's profit under no bundling, and increases also the profits of $B1, B2$ in Γ^s. Hence, for a large α bundling does not deter entry with respect to no bundling, but still entry in Γ^g is less likely than in Γ^s.

[6] Subsections 3.1.1 and 4.1.1 about price competition between firm A and a generalist rival or specialist rivals rely on results from HJM.

specialist firms $B1, B2$, and Γ^g to denote the game in which A faces the generalist firm B.

In either setting, each consumer has unit demand for product j, for $j = 1, 2$. Moreover, product Aj has a higher quality than product Bj, hence it yields a higher utility to consumers than Bj, for $j = 1, 2$. Precisely, each consumer has the same monetary gross utility v_{ij} from consuming product ij, for $i = A, B$, $j = 1, 2$, such that $v_{B1} = v_{B2} = v > 0$ and $v_{A1} = v_{A2} = v + \alpha$ with $\alpha \geq 0$. Thus, α is the utility difference between products Aj and Bj and represents a measure of firm A's dominance over its rival(s).

Firms are also horizontally differentiated à la Hotelling such that relative to product j, consumers are uniformly distributed over the unit interval and firms are located at the interval extremes: firm A is at 0, firm B (or firm Bj) is at 1. A consumer with location $x_j \in [0, 1]$ for product j incurs a transportation cost x_j $(1 - x_j)$ if he buys product Aj (product Bj),[7] that is the cost is equal to the distance between the consumer and the firm offering the product the consumer buys. Hence, for a consumer located at x_j the net utility from buying product Aj (for instance) is $v + \alpha - x_j$ minus the payment to firm A to buy Aj. Moreover, utility is additive over the two products, and we assume that v is large such that each consumer buys $A1$ or $B1$, and $A2$ or $B2$.[8] Finally, the consumers' locations relative to the two products are independently distributed between the two intervals.

For each firm i, let c denote the marginal cost for product ij (for $j = 1, 2$). We assume that c is large enough such that each consumer buys only one unit of product 1, and only one unit of product 2. Then, since marginal costs have an additive effect on prices, without loss of generality we simplify the notation by setting $c = 0$ and interpret prices as profit margins.

3 Competition between firm A and firm B: Game Γ^g

In game Γ^g, the players are firm A and firm B and the timing is as follows:

- Stage one: Firm B chooses between entering and not entering.
- Stage two: After observing firm B's entry decision, A chooses between offering its products separately (independent pricing, IP from now on), and offering them in a bundle (pure bundling, PB from now on).
- Stage three: If B has not entered, then firm A acts as a monopolist. If B has entered, then A and B compete in prices under the pricing regime determined by A at stage two.

[7] We may allow for any marginal transportation cost $t > 0$, but that would have a multiplicative effect on firms' profits which would not change our results qualitatively.

[8] This implies that we can view products 1 and 2 as complements such that in order to obtain a positive utility, each consumer needs one system composed of one product 1 and one product 2.

Notice that if B has entered and A has chosen IP, then at stage three firm A (B) sets the prices p_{A1}, p_{A2} (p_{B1}, p_{B2}) for its single products, and each consumer may buy (for instance) products $A1$ and $B2$, for a total payment of $p_{A1} + p_{B2}$. Conversely, if A has chosen PB then competition occurs between bundles because each consumer either buys the bundle of A, or buys the products $B1, B2$ which in practice are bundled. In this case firm A (B) sets the price P_A (P_B) for the bundle of its products.[9]

In this section we use backward induction to identify the subgame perfect Nash equilibria (SPNE) of Γ^g. Precisely, for each terminal subgame of Γ^g (a subgame that starts at stage three) we identify the unique equilibrium and the equilibrium profits. These are used to deduce A's stage two choice about bundling or not, and finally stage one B's choice about entering or not.

3.1 Stage three in Γ^g

3.1.1 Subgames such that firm B has entered

If firm B has entered, then the stage three competition between A and B is affected by A's choice at stage two.

If competition occurs under IP, then each market can be analyzed in isolation. In market j, a consumer located at x_j buys product Aj if and only if his utility with Aj is higher than with Bj, that is if and only if $v + \alpha - x_j - p_{Aj} \geq v - (1 - x_j) - p_{Bj}$, which is equivalent to

$$x_j \leq \frac{1}{2}(1 + \alpha + p_{Bj} - p_{Aj}) \tag{1}$$

We use $x_j^m = \frac{1}{2}(1 + \alpha + p_{Bj} - p_{Aj})$ to denote the location of the marginal consumer in market j, and F (f) to denote the c.d.f. (the density) for a uniform distribution with support $[0, 1]$. Hence, the demand for product Aj is $F(x_j^m)$; $p_{Aj}F(x_j^m)$ is A's profit in market j, $p_{Bj}(1-F(x_j^m))$ is B's profit in market j, gross of the entry cost.

If competition occurs under PB, then a consumer located at (x_1, x_2) buys A's bundle if and only if $2v + 2\alpha - (x_1 + x_2) - P_A \geq 2v - (1 - x_1 + 1 - x_2) - P_B$, which is equivalent to

$$\bar{x} \leq \frac{1}{2}(1 + \alpha + p_B - p_A) \tag{2}$$

in which $\bar{x} = \frac{x_1+x_2}{2}$ is the consumer's average location, $p_A = \frac{1}{2}P_A$ ($p_B = \frac{1}{2}P_B$) is the average price of the products in A's bundle (in B's bundle). We use $\bar{x}^{gm} = \frac{1}{2}(1 + \alpha + p_B - p_A)$ to denote the average location of the marginal consumer, and comparing (1) and (2) shows that competition under PB is analogous to competition in the market for a single product under

[9] We assume that only firm A decides whether to bundle its products or not, even though also firm B may want to bundle. Allowing B to bundle its products would not change our results, as Lemma 1(iii-iv) below establishes that if B wants to bundle, then also A wants to bundle.

IP, but the distribution of the average location is not uniform. Precisely, the c.d.f. and the density of the average location, denoted with $\bar{F}, \bar{f}$, respectively, are

$$\bar{F}(x) = \begin{cases} 2x^2 & \text{if } 0 \leq x \leq \frac{1}{2} \\ 1-2(1-x)^2 & \text{if } \frac{1}{2} < x \leq 1 \end{cases} \qquad \bar{f}(x) = \begin{cases} 4x & \text{if } 0 \leq x \leq \frac{1}{2} \\ 4(1-x) & \text{if } \frac{1}{2} < x \leq 1 \end{cases} \tag{3}$$

Hence, the demand for the bundle of A is $\bar{F}(\bar{x}^{gm})$; $P_A\bar{F}(\bar{x}^{gm})$ is the profit of A, and $P_B\left(1-\bar{F}(\bar{x}^{gm})\right)$ is the profit of B gross of the entry cost. In the following, we denote with π_i^g (with Π_i^g) the equilibrium profit – upon entry of B – of firm i in Γ^g under IP (under PB), for $i = A, B$. Notice that under IP, firm B obtains a positive market share (in either market) if $\alpha < 3$, but obtains zero market share if $\alpha \geq 3$: in this case each consumer buys from firm A. As a consequence, the equilibrium prices and profit of B are zero if $\alpha \geq 3$. Conversely, under PB firm B's equilibrium market share and profit are positive for each $\alpha \geq 0$.

Lemma 1 (HJM) *In game Γ^g, suppose that firm B has entered. Then*
(i) Under IP, the equilibrium prices and profits for firms A and B are:

$$p^g_{A1} = p^g_{A2} = \max\{1+\frac{1}{3}\alpha, \alpha-1\} \equiv p^*_A, \qquad p^g_{B1} = p^g_{B2} = \max\{1-\frac{1}{3}\alpha, 0\} \equiv p^*_B \tag{4}$$

$$\pi^g_A = \max\{\frac{(3+\alpha)^2}{9}, 2\alpha-2\}, \qquad \pi^g_B = \left(\max\{1-\frac{1}{3}\alpha, 0\}\right)^2 \tag{5}$$

(ii) Under PB, the equilibrium prices and profits for firms A and B are:

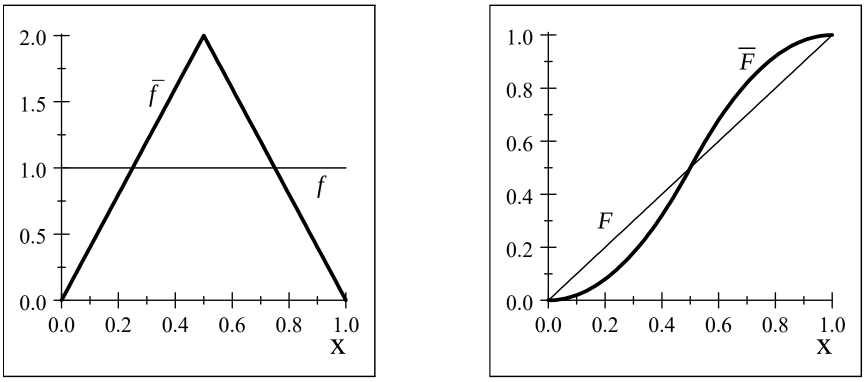
$$P^g_A = \frac{11+2\alpha-\alpha^2+(\alpha-1)\beta^g}{2(1-\alpha+\beta^g)}, \qquad P^g_B = \frac{1-\alpha+\beta^g}{4} \qquad \textit{with} \quad \beta^g = \sqrt{9+\alpha^2-2\alpha}$$

$$\Pi^g_A = \frac{(11+2\alpha-\alpha^2+(\alpha-1)\beta^g)^2}{32(1-\alpha+\beta^g)}, \qquad \Pi^g_B = \frac{(1-\alpha+\beta^g)^3}{128}$$

(iii) If $\alpha \in [0, 1.415)$, then $\pi^g_A > \Pi^g_A$; if $\alpha > 1.415$, then $\pi^g_A < \Pi^g_A$.
(iv) If $\alpha \in [0, 2.376)$, then $\pi^g_B > \Pi^g_B$; if $\alpha > 2.376$, then $\pi^g_B < \Pi^g_B$.

Lemma 1(iii-iv) reveals that the qualitative effect of PB on profits depends on α. As HJM illustrate, this is the result of two effects: the *demand size effect* and the *demand elasticity effect*. Before introducing them, in Figure 1 below we represent graphically $f, \bar{f}$ (in the left panel) and $F, \bar{F}$ (in the right panel).

Figure 1 :



left panel: density for the uniform distribution (thin) and for the average (thick)
right panel: c.d.f. for the uniform distribution (thin) and for the average (thick)

Notice that $\bar{f}$ is symmetric around $\frac{1}{2}$ as f is, but $\bar{f}$ is *more peaked* than f around $\frac{1}{2}$ in the sense that $\int_z^{1-z} f(s)ds < \int_z^{1-z} \bar{f}(s)ds$ for each $z \in (0, \frac{1}{2})$, that is $\bar{f}$ puts more weight around $\frac{1}{2}$ than f. This implies

$$\bar{F}(x) > F(x) \qquad \text{for each} \quad x \in (\frac{1}{2}, 1).$$

For the demand size effect, consider competition under IP and the equilibrium prices p_A^* and p_B^* in (4). Then the location of the marginal consumer in each market is $x^* = \frac{1}{2}(1+\alpha+p_B^*-p_A^*)$, and for $\alpha \in (0,3)$ we have that $x^* \in (\frac{1}{2}, 1)$.[10] Now consider competition under PB with $P_A = 2p_A^*$ and $P_B = 2p_B^*$, that is the average price for each product in the bundle of firm i is p_i^*. Then the average location of the marginal consumer is still x^*, hence the demand for firm A changes from $F(x^*)$ for each of its products to $\bar{F}(x^*)$ for its bundle, and $\bar{F}(x^*) > F(x^*)$ holds since $x^* \in (\frac{1}{2}, 1)$. Therefore, bundling with unchanged unit prices increases the demand for firm A, reduces the demand for B. This is the demand size effect, and it is weak if x^* is close to $\frac{1}{2}$ or close to 1, but is strong if x^* is about intermediate in the interval $(\frac{1}{2}, 1)$.

The demand elasticity effect affects the firms' incentives to modify prices under PB with respect to $P_A = 2p_A^*$, $P_B = 2p_B^*$. We find that firm B, given $P_A = 2p_A^*$, always has incentive to reduce P_B below $2p_B^*$ because its demand under bundling is more elastic. This occurs mainly because the market share reduction of firm B due to the demand size effect makes B willing to give up some profit on its inframarginal consumers in exchange for a higher market share.[11] Conversely, the elasticity of demand of firm A under PB depends on α. Precisely, for α close to zero x^* is close to $\frac{1}{2}$ and this makes the demand of A more elastic under PB (like for B) as $\bar{F}(x^*)$ and $F(x^*)$ are both about equal to $\frac{1}{2}$ and $\bar{f}(x^*) > f(x^*)$ implies that a given decrease in

[10] We consider the case of $\alpha \geq 3$ (which implies $x^* = 1$) at the end of this subsection.

[11] In fact, bundling also modifies how demand reacts to a price change, sometimes increasing and sometimes decreasing demand reactivity. But even when demand reactivity decreases, the reduction in demand generated by the demand size effect prevails and increases the elasticity of demand for firm B.

the average price of the bundle generates a higher increase in demand than the same decrease in the prices of the single products. When α is high, the opposite occurs for A: its demand is less elastic under PB because when x^* is close to 1, $\bar{F}(x^*)$ and $F(x^*)$ are both close to 1 and $f(x^*) = 1$ is greater than $\bar{f}(x^*)$, which is close to 0. For intermediate values of α, the change in the elasticity for firm A is small, hence A has weaker incentive to change the price than in the above extreme cases.

Summarizing, if α is small (precisely, if $\alpha < 1.415$) then the demand size effect is weak but the demand elasticity effect intensifies competition and makes both firms worse off under PB than under IP. For intermediate α ($1.415 < \alpha < 2.376$) there is a strong demand size effect, positive for A and negative for B, which dominates the demand elasticity effect and makes A better off and B worse off under PB. Finally, if α is large ($\alpha > 2.376$) then the demand size effect is weak but demand is less elastic for A, which induces A to charge a higher price under PB. This leaves room to firm B to increase the price of its bundle and make a higher profit under PB than under IP, such that both firms are better off under PB. In particular, if $\alpha \geq 3$ then $x^* = 1$ and there is no demand size effect, but the demand elasticity effect works as described just above.

3.1.2 Subgames such that firm B has not entered

If firm B has not entered, then A is monopolist. Under IP, in market j a consumer located at x_j buys product Aj if and only if his utility from the purchase is non-negative, that is if $v + \alpha - x_j - p_{Aj} \geq 0$. Hence, the demand for this product is $F(v + \alpha - p_{Aj})$. Since v is large, the optimal p_{Aj} for firm A is $v + \alpha - 1$, the lowest possible consumer valuation for product Aj, which induces each consumer to buy Aj. Any $p_{Aj} > v + \alpha - 1$ is unprofitable for A as $f(1) = 1$ and v large imply that the revenue lost from consumers who stop buying Aj exceeds the revenue increase from the consumers that still buy Aj.

Now suppose that A has chosen PB at stage two. A consumer located at (x_1, x_2) buys the bundle of A if and only if $2v + 2\alpha - x_1 - x_2 - P_A \geq 0$, that is $v + \alpha - p_A \geq \bar{x}$ with $\bar{x} = \frac{1}{2}(x_1 + x_2)$ and $p_A = \frac{1}{2}P_A$. Hence, the demand for A's bundle is $\bar{F}(v + \alpha - p_A)$.

Lemma 2 *(i) If firm A is monopolist under IP, then the optimal price in market j is $p_{Aj} = v + \alpha - 1$, for $j = 1, 2$, and A's total profit is $2(v + \alpha - 1)$.*
(ii) If firm A is monopolist under PB, then the optimal price for the bundle is slightly greater than $2v + 2\alpha - 2$, and A's profit is greater than $2v + 2\alpha - 2$.

About Lemma 2(ii), notice that $p_A = v + \alpha - 1$ under PB is equivalent to $p_{A1} = p_{A2} = v + \alpha - 1$ under IP, as it induces each consumer to buy A's bundle and yields A a profit of $2v + 2\alpha - 2$, equal to A's highest profit under IP. However, A can do better by slightly increasing p_A above $v + \alpha - 1$ (i.e., by increasing P_A above $2v + 2\alpha - 2$) because that increases the revenue from the consumers that continue to buy the bundle and makes A lose very few consumers. Formally, the demand elasticity for the bundle is $\frac{-p_A \bar{f}(v+\alpha-p_A)}{\bar{F}(v+\alpha-p_A)}$, and it is 0 at $p_A = v+\alpha-1$ because $\bar{f}(1) = 0$. Hence, a small increase in p_A above $v+\alpha-1$ is profitable because it reduces A's demand by a negligible amount.[12] In the case of IP, this argument does not work because $f(1) = 1 \neq 0$; hence increasing p_{A1} and/or p_{A2} above $v+\alpha-1$ makes firm A lose a non-negligible amount of consumers.

3.2 Stage two in Γ^g

From Lemmas 1(iii) and 2 we deduce firm A's choice at stage two.

Lemma 3 *In game* Γ^g
(i) if firm B has entered at stage one, then A's best action at stage two is IP if $\alpha \in [0, 1.415)$*, is PB if* $\alpha > 1.415$*;*
(ii) if firm B has not entered, then A's best action at stage two is PB.

3.3 Stage one in Γ^g

Here we study the entry decision of firm B, which is based on anticipating the equilibrium play at stages two and three. Recall that if B enters, then it incurs an entry cost of $2k$. When $\alpha < 1.415$, B knows that A will choose IP in case of entry; hence B compares π_B^g with $2k$. When $\alpha > 1.415$, A will choose PB if B enters and B compares Π_B^g with $2k$. This yields Proposition 1.

Proposition 1 *In game* Γ^g*, the unique SPNE is such that*
(i) when $0 \leq \alpha < 1.415$*, at stage one firm B enters if and only if* $k < \frac{1}{2}\pi_B^g$*;*
(ii) when $1.415 < \alpha$*, at stage one firm B enters if and only if* $k < \frac{1}{2}\Pi_B^g$*;*
for each α*, the rest of the SPNE strategies (relative to stages two and three) is obtained from Lemmas 1-3.*

4 Competition among firm A and firms $B1, B2$: Game Γ^s

In game Γ^s, the players are firms A and $B1, B2$ (we may think that B has been split into the two firms $B1, B2$), and the timing is as follows:

- Stage one: Firms $B1$ and $B2$ simultaneously and non-cooperatively decide whether to enter or not.

[12] This is an application of the exclusion principle described in Armstrong (1996). After establishing Lemma 2(i), it is possible to prove that A prefers PB to IP by applying Proposition 4 in Fang and Norman (2006).

- Stage two: After observing the entry decisions of $B1$ and $B2$, firm A chooses between IP and PB.
- Stage three: If no specialist firm has entered, then A acts as a monopolist. If $B1$ and/or $B2$ has entered, then the active firms compete in prices under the pricing regime determined by A's stage two choice.

If $B1, B2$ have entered and A has chosen IP, then competition occurs essentially like in game Γ^g, except that firm $B1$ ($B2$) sets p_{B1} (p_{B2}). If instead A has chosen PB, then competition occurs between the bundle of A and the bundle of products $B1\&B2$, and firm A sets P_A, firms $B1, B2$ choose p_{B1}, p_{B2}, hence $p_{B1} + p_{B2}$ is the price of the bundle $B1\&B2$. As for Γ^g, we apply backward induction to Γ^s and begin with the analysis of the terminal subgames.

4.1 Stage three in Γ^s

4.1.1 Subgames such that $B1$ and $B2$ both have entered

If competition occurs under IP after both $B1$ and $B2$ have entered, then the analysis of Subsection 3.1.1 still applies. Precisely, there are no links between the market for product 1 and the market for product 2, hence it does not matter whether A faces the separate entities $B1$, $B2$ or a merged single entity B, except that in the former case the equilibrium profit of $B1$ ($B2$) if half of the equilibrium profit of B in the latter case. In order to simplify notation for the rest of the paper, we define $\hat{\pi} = \frac{1}{2}\pi_B^g = \frac{1}{2}\left(\max\{1 - \frac{1}{3}\alpha, 0\}\right)^2$, hence $\hat{\pi}$ is the profit of both $B1$ and $B2$ under IP. Also notice that using $\hat{\pi}$, Proposition 1(i) can be stated as follows: When $0 \leq \alpha < 1.415$, the unique SPNE is such that firm B enters if and only if $k < \hat{\pi}$.

If competition occurs under PB, then we can use (2) to see that a consumer located at (x_1, x_2) buys A's bundle if and only if his average location $\bar{x}$ satisfies

$$\bar{x} \leq \frac{1}{2}(1 + \alpha + \frac{1}{2}p_{B1} + \frac{1}{2}p_{B2} - \frac{1}{2}P_A)$$

Hence, $\bar{x}^{sm} = \frac{1}{2}(1 + \alpha + \frac{1}{2}p_{B1} + \frac{1}{2}p_{B2} - \frac{1}{2}P_A)$ is the average location of the marginal consumer, the demand for the bundle of A is $\bar{F}(\bar{x}^{sm})$, A's profit is $P_A\bar{F}(\bar{x}^{sm})$, and the profits for firms $B1$, $B2$ (gross of the entry costs) are $p_{B1}[1 - \bar{F}(\bar{x}^{sm})]$, $p_{B2}[1 - \bar{F}(\bar{x}^{sm})]$, respectively. We use π_i^s (Π_i^s) to denote the equilibrium profit of firm i upon entry of $B1, B2$ under IP (under PB).

Lemma 4 (HJM) *In game Γ^s, suppose that firms $B1$ and $B2$ have entered. Then*
(i) Under IP, the equilibrium prices and profits for firms A and $B1, B2$ are:

$$p_{A1}^s = p_{A2}^s = p_A^*, \qquad p_{B1}^s = p_{B2}^s = p_B^*$$

$$\pi_A^s = \max\{\frac{(3+\alpha)^2}{9}, 2\alpha - 2\} = \pi_A^g, \qquad \pi_{B1}^s = \pi_{B2}^s = \hat{\pi}$$

(ii) Under PB, the equilibrium prices and profits for firms A and $B1, B2$ are:

$$P_A^s = \frac{2(19 - \alpha^2 + 2\alpha + (\alpha - 1)\beta^s)}{5(1 - \alpha + \beta^s)}, \qquad p_{B1}^s = p_{B2}^s = \frac{1 - \alpha + \beta^s}{5} \qquad \textit{with } \beta^s = \sqrt{11 + \alpha^2 - 2\alpha}$$

$$\Pi_A^s = \frac{2(19 - \alpha^2 + 2\alpha + (\alpha - 1)\beta^s)^2}{125(1 - \alpha + \beta^s)}, \qquad \Pi_{B1}^s = \Pi_{B2}^s = \frac{(1 - \alpha + \beta^s)^3}{250}$$

(iii) If $\alpha \in [0, 0.307)$, then $\pi_A^s > \Pi_A^s$; if $\alpha > 0.307$, then $\pi_A^s < \Pi_A^s$.
(iv) If $\alpha \in [0, 2.092)$, then $\hat{\pi} > \Pi_{B1}^s$; if $\alpha > 2.092$, then $\hat{\pi} < \Pi_{B1}^s$.

Comparing Lemma 1(iii-iv) with Lemma 4(iii-iv) reveals that in Γ^s it is more frequent that PB has a positive effect on firms' profits. In particular, for firm A the inequality $\Pi_A^g > \pi_A^g$ holds for $\alpha > 1.415$, whereas $\Pi_A^s > \pi_A^s$ holds more frequently, for $\alpha > 0.307$. This difference arises because in the competition under PB, firms $B1, B2$ charge a higher total price than firm B. Precisely, for each given P_A, if firm $B1$ reduces p_{B1} then the demand for the bundle $B1\&B2$ increases, but firm $B1$ does not take into account that this has a positive effect on the profit of $B2$ and chooses p_{B1} just to maximize the own profit. Conversely, an integrated firm B would take this externality into account and charge a lower total price than the total price charged by $B1, B2$.[13] Hence, as HJM remark, under PB firm A prefers to compete against $B1, B2$ rather than against B because competition against $B1, B2$ is softer:

$$\Pi_A^s > \Pi_A^g \qquad \text{for each } \alpha \geq 0 \tag{6}$$

Moreover, it turns out that also $B1$ and $B2$ are collectively better off in Γ^s than firm B in Γ^g. The reason is that $p_{B1} + p_{B2}$ higher than P_B makes it profitable for A to increase P_A, as prices are strategic complements. We find that in our setting this effect dominates the lack of coordination between $B1$ and $B2$ and implies $\Pi_{B1}^s + \Pi_{B2}^s > \Pi_B^g$, or equivalently, since $\Pi_{B1}^s = \Pi_{B2}^s$,

$$\Pi_{B1}^s > \frac{1}{2}\Pi_B^g \qquad \text{for each } \alpha \geq 0 \tag{7}$$

This explains why for firm B the inequality $\hat{\pi} < \frac{1}{2}\Pi_B^g$ holds for $\alpha > 2.376$, but $\hat{\pi} < \Pi_{B1}^s$ holds more frequently, for $\alpha > 2.092$. In particular, (6)-(7) establish that for each firm competition under PB is more profitable in Γ^s than in Γ^g.

4.1.2 Subgames such that only one specialist firm has entered

Suppose that only one specialist firm has entered, say firm $B1$ to fix the ideas. If A has chosen IP at stage two, then A faces competition in market 1 but is monopolist in market 2. Then the equilibrium prices are described by Lemma 1(i) for market 1, by Lemma 2(i) for market 2.

[13] This effect is pointed out by Denicolò (2000) and Nalebuff (2000) in related models.

Matters are more complicated if competition occurs under PB, because then each consumer either buys A's bundle or buys the single product $B1$. Precisely, a consumer located in (x_1, x_2) buys the bundle of A if and only if $2v + 2\alpha - x_1 - x_2 - P_A \geq v - (1 - x_1) - p_{B1}$, which is equivalent to $x_2 \leq -2x_1 + 1 + w + p_{B1} - P_A$ with $w = v + 2\alpha$. Therefore, the demand for A's bundle is equal to 1 if $P_A - p_{B1} < w - 2$, is equal to 0 if $P_A - p_{B1} > w + 1$, and if $w - 2 \leq P_A - p_{B1} \leq w + 1$ it is

$$D_A(P_A, p_{B1}) = \begin{cases} 1 - \frac{1}{4}(2 - w - p_{B1} + P_A)^2 & \text{if } w - 2 \leq P_A - p_{B1} \leq w - 1 \\ \frac{1}{4}(2w + 2p_{B1} - 2P_A + 1) & \text{if } w - 1 < P_A - p_{B1} \leq w \\ \frac{1}{4}(1 + w + p_{B1} - P_A)^2 & \text{if } w < P_A - p_{B1} \leq w + 1 \end{cases}$$

The demand for product $B1$ is $1 - D_A(P_A, p_{B1})$ and the profit functions are $P_A D_A(P_A, p_{B1})$, $p_{B1}[1 - D_A(P_A, p_{B1})]$ for A and for $B1$, respectively. We use $\pi_A^{s1}, \pi_{B1}^{s1}$ $(\Pi_A^{s1}, \Pi_{B1}^{s1})$ to denote the equilibrium profits under IP (under PB) when only $B1$ enters.[14]

Lemma 5 *In game Γ^s, suppose that only firm $B1$ has entered and that $w > 2$. Then*
(i) Under IP, the equilibrium prices and profits for firms A and $B1$ are

$$p_{A1}^{s1} = p_A^*, \qquad p_{A2}^{s1} = v + \alpha - 1, \qquad p_{B1}^{s1} = p_B^*$$

$$\pi_A^{s1} = \max\{\frac{(3+\alpha)^2}{18}, \alpha - 1\} + v + \alpha - 1, \qquad \pi_{B1}^{s1} = \hat{\pi}$$

(ii) Under PB, the equilibrium prices and profits for firms A and $B1$ are

$$P_A^{s1} = \frac{5}{8}w - \frac{5}{4} + \frac{3}{8}\beta^{s1}, \qquad p_{B1}^{s1} = \frac{1}{8}\beta^{s1} - \frac{1}{8}w + \frac{1}{4} \qquad \text{with} \qquad \begin{array}{c} w = v + 2\alpha \\ \beta^{s1} = \sqrt{(w-2)^2 + 16} \end{array}$$

$$\Pi_A^{s1} = w - 2 + \frac{4w^2 - 16w + 178}{(w^2 - 4w + 40)(\beta^{s1} + w - 2) + 8w - 16}$$

$$\Pi_{B1}^{s1} = \frac{2}{(w^2 - 4w + 8)(\beta^{s1} + w - 2) + 8w - 16}$$

(iii) For each $\alpha \in [0, 3)$, $\pi_A^{s1} > \Pi_A^{s1}$ and $\pi_{B1}^{s1} > \Pi_{B1}^{s1}$ hold provided that v is sufficiently large, but $\pi_A^{s1} < \Pi_A^{s1}$ and $\pi_B^{s1} < \Pi_B^{s1}$ if $\alpha \geq 3$.

We can use again the demand size effect and the demand elasticity effect to explain the results in Lemma 5(iii). When $\alpha < 3$, moving from IP to PB with $P_A = p_{A1}^{s1} + p_{A2}^{s1}$, $p_{B1} = p_{B1}^s$ makes firm A gain some market share in market 1 and lose market share in market 2. Although A's total market share increases, the effect on the profit of A is negative since a large v makes p_{A2}^{s1} much larger than p_{A1}^{s1}, and then the sales lost in market 2 are not compensated by the

[14] The analysis of this pricing game when $\alpha = 0$ appears in a previous version of HJM.

sales gained in market 1.[15] Thus the demand size effect reduces A's profit and also $B1$'s profit because firm $B1$ loses market share in market 1, the unique market in which $B1$ is active.

In addition, the demand elasticity effect induces both firms to reduce prices. For firm $B1$, the reason is similar to the one described in Subsection 3.1.1, as a lower market share reduces the loss on inframarginal consumers from reducing p_{B1}.[16] For firm A it is convenient to reduce P_A in order to recover market share on the sale of the very profitable product $A2$. Therefore, competition between A's bundle and product $B1$ becomes more intense, which contributes to reducing the profit of each firm with respect to IP. This is similar to the result in Whinston (1990) mentioned in the introduction, in which a multiproduct firm faces a threat of entry in a single market and bundling, upon entry, reduces the entrant's profit and also the profit of the bundling firm.

For $\alpha \geq 3$, the opposite result holds and both firms prefer PB, like in game Γ^g when $\alpha \geq 3$. Precisely, since firm A has full market share in both markets under IP, there is no demand size effect. But under PB it is profitable for A to raise the price of the bundle slightly above the sum $p_{A1}^{s1} + p_{A2}^{s1}$ because its demand elasticity is zero given $P_A = p_{A1}^{s1} + p_{A2}^{s1}$ and $p_{B1} = 0$. This allows $B1$ to charge a (small) positive price and make some sales; thus $B1$ makes a positive profit under PB.

4.1.3 Subgames such that neither specialist has entered

In this case Lemma 2 applies.

4.2 Stage two in Γ^s

Using Lemmas 2, 4(iii), 5(iii), we deduce A's choice at stage two.

Lemma 6 *In game* Γ^s
(i) if $B1$ *and* $B2$ *have entered, then at stage two* A*'s best action is IP if* $\alpha \in [0, 0.307)$*, is PB if* $\alpha > 0.307$*;*
(ii) if only one specialist has entered, then IP is A*'s best action at stage two if* $\alpha < 3$*, PB is* A*'s best action if* $\alpha \geq 3$*;*
(iii) if no specialist has entered, then PB is A*'s best action at stage two.*

4.3 Stage one in Γ^s

Here we examine the entry decisions of $B1, B2$ at stage one. These decisions are affected by the expectations of $B1$ and $B2$ about A's choice at stage two, which depends on α as described

[15] In Subsections 3.1.1 and 4.1.1 the demand size effect increases/decreases firm A's profit if and only if it increases/decreases firm A's total market share because $p_{A1}^g = p_{A2}^g$ and $p_{A1}^s = p_{A2}^s$.

[16] In fact, under PB the reactivity of demand for product $B1$ with respect to p_{B1} is smaller, but the demand reduction described above dominates and increases the demand elasticity for product $B1$.

by Lemma 6. Hence, we distinguish the case of $\alpha \in [0, 0.307)$ from the case of $\alpha > 0.307$,[17] and recall that each specialist incurs a cost k in case of entry. Moreover, from Lemmas 4-5 we see that no specialist enters if $k > \max\{\hat{\pi}, \Pi^s_{B1}\}$.

Case of $\alpha \in [0, 0.307)$ When $\alpha \in [0, 0.307)$, A chooses IP if $B1, B2$ have entered. Hence, the game at stage one between $B1$ and $B2$ has the following normal form, in which a firm enters (stays out) by playing E (NE):

$B1 \backslash B2$	E	NE
E	$\hat{\pi} - k, \hat{\pi} - k$	$\hat{\pi} - k, 0$
NE	$0, \hat{\pi} - k$	$0, 0$

(8)

In this game there is no strategic interaction between $B1$ and $B2$ as each firm's profit is independent of the action of the other firm. This occurs because under IP, there is no link between the markets for the two products, hence for a specialist it does not matter whether the other specialist enters or not. Then for each player a strictly dominant strategy exists, unless $\hat{\pi} - k = 0$. In particular, E is strictly dominant if $k < \hat{\pi}$ and in that case (E, E) is the unique equilibrium of (8). The rest of the SPNE strategies (relative to stages two and three) is obtained from Lemmas 2 and 4-6.

Proposition 2 *In game Γ^s, suppose that $\alpha \in [0, 0.307)$. Then the unique SPNE is such that firms $B1$ and $B2$ both enter if $k < \hat{\pi}$, neither $B1$ nor $B2$ enters if $\hat{\pi} < k$.*

Case of $\alpha > 0.307$ When $0.307 < \alpha < 3$, firm A chooses PB if $B1, B2$ enter. Hence, the game at stage one between $B1$ and $B2$ has the following normal form:

$B1 \backslash B2$	E	NE
E	$\Pi^s_{B1} - k, \Pi^s_{B1} - k$	$\hat{\pi} - k, 0$
NE	$0, \hat{\pi} - k$	$0, 0$

(9)

We need to distinguish two subcases: The first one is such that $\Pi^s_{B1} < \hat{\pi}$, that is $\alpha \in (0.307, 2.092)$; the second one is such that $\Pi^s_{B1} > \hat{\pi}$, that is $2.092 < \alpha < 3$. When $\Pi^s_{B1} < \hat{\pi}$, a specialist firm that enters is better off if the other specialist does not enter, whereas if $\Pi^s_{B1} > \hat{\pi}$ then an entrant specialist prefers that also the other specialist enters. These differences affect the structure of the equilibria of (9).

In the first case, if $k < \Pi^s_{B1} < \hat{\pi}$ then E is strictly dominant in (9) for each player and (E, E) is the unique equilibrium. Conversely, if $\Pi^s_{B1} < k < \hat{\pi}$ then (9) is an hawk-dove game

[17] By Lemma 6(ii), we also need to distinguish $\alpha < 3$ from $\alpha \geq 3$, but this has a small effect as illustrated by Footnote 18.

with two asymmetric equilibria, (NE, E) and (E, NE), and the following symmetric mixed strategy equilibrium:

$$\text{firm } B1\ (B2) \text{ enters with probability } p = \frac{\hat{\pi} - k}{\hat{\pi} - \Pi^s_{B1}}, \text{ stays out with probability } 1 - p \qquad (10)$$

Since (9) is a symmetric game, we assume that $B1$ and $B2$ play the equilibrium (10) when $\Pi^s_{B1} < k < \hat{\pi}$.

In the second case, if $k < \hat{\pi} < \Pi^s_{B1}$ then again E is strictly dominant in (9) for each player and (E, E) is the unique equilibrium. But if $\hat{\pi} < k < \Pi^s_{B1}$ then (9) is a coordination game with three equilibria: (E, E), (NE, NE), and the mixed strategy equilibrium (10). In this case all the equilibria are symmetric, and then we use Pareto dominance as a refinement. Since (E, E) Pareto dominates the other two equilibria, we suppose that $B1$ and $B2$ play (E, E) when $\hat{\pi} < k < \Pi^s_{B1}$.[18] This yields next proposition about game (9), and the rest of the SPNE strategies (relative to stages two and three) is obtained from Lemmas 2 and 4-6.

Proposition 3 *In game Γ^s, suppose that $\alpha > 0.307$. Then neither $B1$ nor $B2$ enters if $k > \max\{\hat{\pi}, \Pi^s_{B1}\}$. If instead $k < \max\{\hat{\pi}, \Pi^s_{B1}\}$, then the stage one entry game is such that*
(i) for $\alpha \in (0.307, 2.092)$, (E, E) is the unique equilibrium if $k < \Pi^s_{B1}$; the unique symmetric equilibrium is (10) if $\Pi^s_{B1} < k < \hat{\pi}$;
(ii) for $\alpha > 2.092$, (E, E) is the unique equilibrium if $k < \hat{\pi}$; the unique Pareto dominant equilibrium is (E, E) if $\hat{\pi} < k < \Pi^s_{B1}$.

Therefore, both specialists enter if $k < \Pi^s_{B1}$, but if $\Pi^s_{B1} < k$ then a specialist enters with positive probability only if $k < \hat{\pi}$.

5 Comparison between Γ^g and Γ^s

In this section we compare the outcomes of Γ^g and Γ^s in terms of social welfare, consumers' surplus, and firms' profits. Precisely, for firms B and $B1, B2$ we compare the profit of B in Γ^g with the sum of the profits of $B1$ and $B2$ in Γ^s; in a sense, this inquires the profitability for firm B of splitting into two distinct firms.

First notice that in some cases Γ^g and Γ^s lead to the same outcome. Precisely, if $\alpha \in (0, 0.307)$ then A chooses IP whenever entry occurs (Lemmas 3(i) and 6(i-ii)), which implies that in Γ^g firm B enters if and only if $k < \hat{\pi}$, in Γ^s firms $B1, B2$ both enter if and only if $k < \hat{\pi}$ (Propositions 1 and 2). Therefore, in both games firm A is monopolist if $\hat{\pi} < k$, whereas it faces competition in both markets, under IP, if $k < \hat{\pi}$. But in fact the same outcome arises

[18] If $\alpha \geq 3$, then $\hat{\pi}$ in (9) should be replaced by Π^{s1}_{B1} (by Lemma 6(ii)) but Π^{s1}_{B1} is close to zero given that v (and w) is large, hence it is still true that if $k < \Pi^s_{B1}$, then (E, E) is the unique equilibrium or is the unique Pareto dominant equilibrium for game (9).

when $k < \hat{\pi}$ since under IP it does not make any difference if A competes against a generalist or against two specialists (Lemmas 1(i) and 4(i)). We conclude that the two games have the same outcome if A never bundles. There is no difference between Γ^g and Γ^s also when $\alpha > 0.307$ and k is greater than $\max\{\hat{\pi}, \Pi^s_{B1}\}$, because then the entry cost is so high that in either game no entry occurs, hence A turns out to be a monopolist both in Γ^g as in Γ^s. Therefore, in the rest of this section we focus on the case in which $\alpha > 0.307$ and $k < \max\{\hat{\pi}, \Pi^s_{B1}\}$.[19]

5.1 Profits' comparison

Since we consider $\alpha > 0.307$ and $k < \max\{\hat{\pi}, \Pi^s_{B1}\}$, it is useful to notice that (by Lemma 4(iv))

$$\max\{\hat{\pi}, \Pi^s_{B1}\} = \begin{cases} \hat{\pi} & \text{if } \alpha \in (0.307, 2.092) \\ \Pi^s_{B1} & \text{if } \alpha > 2.092 \end{cases} \tag{11}$$

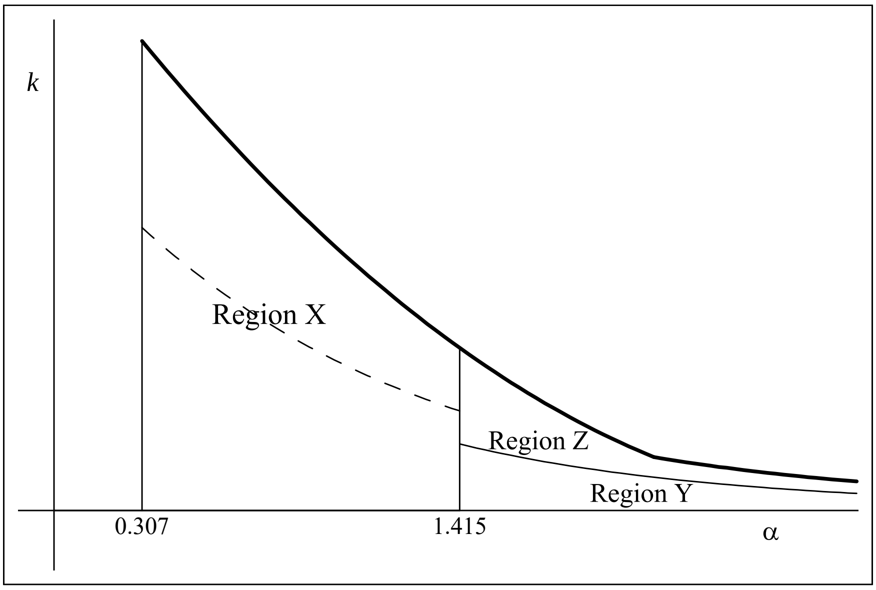
The thick curve in Figure 2 below is the graph of $k = \max\{\hat{\pi}, \Pi^s_{B1}\}$.

Given that $\alpha > 0.307$, in Γ^s firm A chooses PB if $B1, B2$ both enter, but in Γ^g, after B's entry, A chooses IP if $\alpha < 1.415$, PB if $\alpha > 1.415$. We start examining the case of $\alpha < 1.415$, thus we consider the region in the space (α, k) which is denoted Region X in Figure 2; the dashed curve in this region is the graph of $k = \Pi^s_{B1}$, which lies below the thick curve because of (11). We first take k smaller than Π^s_{B1}, hence $k < \hat{\pi}$. In this case, in Γ^g firm B enters and in Γ^s firms $B1, B2$ enter, but competition occurs under IP in Γ^g and A (B) earns π^g_A (earns $2\hat{\pi} - 2k$), competition occurs under PB in Γ^s and A ($B1, B2$) earns Π^s_A (earns $\Pi^s_{B1} - k$). Therefore, A prefers Γ^s but B prefers Γ^g by Lemma 4(iii-iv). Basically, as we described in Subsection 4.1.1, competition under bundling is softer in Γ^s than in Γ^g, and this makes PB more profitable for A than competition under IP for each $\alpha \in (0.307, 1.415)$. Moreover, PB reduces the profit of $B1, B2$ since $\hat{\pi} > \Pi^s_{B1}$, and this reduction is especially relevant if k is between Π^s_{B1} and $\hat{\pi}$ (the points in Region X which are above the dashed curve) because then entry in Γ^s is affected. For $B1, B2$ it becomes unprofitable to enter jointly (see (9)), and by Proposition 3(i) they enter with probability smaller than one, according to (10). Thus, when $\alpha \in (0.307, 1.415)$ firm A is better off in Γ^s both because PB is more profitable than the IP firm A would select in Γ^g, but also because PB reduces the profit of $B1, B2$ from entering. If $k > \Pi^s_{B1}$, this makes each specialist enter with probability less than one, hence A becomes monopolist in one market – or both – with positive probability; the latter outcomes are better for A than facing $B1$ and $B2$ in Γ^s, hence better than competing against B in Γ^g. Therefore, PB is effective in this case in creating an entry barrier against $B1$ and $B2$. In fact, also in Γ^g bundling has the effect of reducing B's profit (and in a stronger way than with $B1, B2$), in some cases below the entry cost, but it is unprofitable for A to play PB after B's entry (Lemma 3(i)). In the jargon of game theory, a threat of A to play PB to discourage entry of B is not credible. Hence, B

[19] Notice that $\tilde{\alpha}, \hat{\alpha}$ mentioned in the introduction are equal to 0.307 and to 1.415, respectively.

expects that A plays IP and views entry as profitable as long as $k < \hat{\pi}$ (Proposition 1(i)).

Figure 2
solid thick curve: $\max\{\hat{\pi}, \Pi^s_{B1}\}$
dashed curve: Π^s_{B1}
solid thin curve: $\frac{1}{2}\Pi^g_B$
:



Matters are different when $\alpha > 1.415$ because then in Γ^g firm A chooses PB after B's entry, hence B enters if and only if $k < \frac{1}{2}\Pi^g_B$ (Proposition 1(ii)); the solid thin curve in Figure 2 is the graph of $k = \frac{1}{2}\Pi^g_B$. Conversely, in Γ^s firms $B1, B2$ both enter as long as $k < \Pi^s_{B1}$ (Proposition 3). Since $\frac{1}{2}\Pi^g_B < \Pi^s_{B1}$ from (7), we conclude that if $k < \frac{1}{2}\Pi^g_B$ then A faces competition in both markets and chooses PB both in Γ^g and in Γ^s. In Figure 2, Region Y is the set of (α, k) which satisfy $\alpha > 1.415$ and $k < \frac{1}{2}\Pi^g_B$. Inequalities (6) and (7) show that A and B both prefer Γ^s in Region Y because competition under bundling is softer in Γ^s than in Γ^g.

The final case to consider is represented by Region Z, the set of (α, k) such that $\alpha > 1.415$ and $\frac{1}{2}\Pi^g_B < k < \max\{\hat{\pi}, \Pi^s_{B1}\}$. In this case B does not enter in Γ^g because its gross profit from entry, Π^g_B, is smaller than its entry cost $2k$; thus Γ^g leads to A's monopoly. Conversely, in Γ^s both $B1$ and $B2$ enter with positive probability, thus A is less often monopolist in Γ^s than in Γ^g, which makes A prefer Γ^g to Γ^s and B prefer Γ^s. Recall from Lemmas 1(iv) and 4(iv) that bundling reduces the profit of A's rivals if $\alpha < 2.092$, hence when $\alpha \in (1.415, 2.092)$ we may claim that PB is more effective in deterring B's entry in Γ^g than the entry of $B1, B2$ in Γ^s. For $\alpha > 2.092$, bundling actually favors entry in Γ^s (and in Γ^g if $\alpha > 2.376$) with respect to IP, but entry remains more difficult in Γ^g than in Γ^s.

Next proposition summarizes the three cases we have considered.

Proposition 4 *Suppose that* $\alpha > 0.307$ *and* $k < \max\{\hat{\pi}, \Pi^s_{B1}\}$*, otherwise* Γ^g *and* Γ^s *lead to the same outcome. Then*
(i) A prefers Γ^s*, B prefers* Γ^g *if* $\alpha \in (0.307, 1.415)$ *(Region X in Figure 2);*
(ii) A and B both prefer Γ^s *if* $\alpha > 1.415$ *and* $k < \frac{1}{2}\Pi^g_B$ *(Region Y in Figure 2);*
(iii) A prefers Γ^g*, B prefers* Γ^s *if* $\alpha > 1.415$ *and* $\frac{1}{2}\Pi^g_B < k$ *(Region Z in Figure 2).*

It is interesting to see Proposition 4 from the point of view of firm B. First notice that for $\alpha \in (0.307, 1.415)$, the profit of firm B (of $B1, B2$) is lower under PB than under IP. Firm A chooses PB in Γ^s, but in Γ^g firm A prefers IP in order to avoid the fierce competition that takes place in Γ^g under PB. Thus firm B would be worse off than $B1, B2$ given competition under PB, but B avoids this tough environment because also A wants to stay out of it. Conversely, firm A is willing to compete against $B1, B2$ under PB and this harms $B1, B2$, reducing their participation. In this case A's possibility to bundle helps to keep A's rival(s) out in Γ^s, but not in Γ^g.

The above arguments do not apply for $\alpha > 1.415$, because then PB is profitable for A also in Γ^g. In this case B cannot avoid competition under PB in Γ^g, and there exists a parameter space in which the credible threat of bundling deters the entry of B but not of $B1, B2$. In this case bundling is more effective as a foreclosure instrument in Γ^g than in Γ^s.

5.2 Comparison in terms of social welfare and consumers' surplus

In order to compare social welfare in Γ^g and in Γ^s, we first illustrate how it is derived. Suppose that firm B enters in Γ^g, and that $B1, B2$ enter in Γ^s. If competition occurs under IP in either game and x is the location of the marginal consumer in each market, then social welfare is $W(x) = 2\,(v - k + \alpha F(x) - T(x))$ in which, for each market, $\alpha F(x)$ represents the higher utility generated by the product of firm A and $T(x) = \int_0^x z f(z)dz + \int_x^1 (1-z) f(z)dz$ is the average transportation cost. If competition takes place under PB and $\bar{x}$ is the average location of the marginal consumer, then social welfare is $\bar{W}(\bar{x}) = 2\left(v - k + \alpha \bar{F}(\bar{x}) - \bar{T}(\bar{x})\right)$, with $\bar{T}(\bar{x}) = \int_0^{\bar{x}} z \bar{f}(z)dz + \int_{\bar{x}}^1 (1-z)\bar{f}(z)dz$. Both W and $\bar{W}$ are single peaked and have the same maximum point $x' = \min\{\frac{1}{2} + \frac{1}{2}\alpha, 1\}$. In particular, $x' > \frac{1}{2}$ as $-T$ in W (and $-\bar{T}$ in $\bar{W}$) is maximized at $x = \frac{1}{2}$, but αF in W (and $\alpha \bar{F}$ in $\bar{W}$) is increasing in x. If $\alpha \geq 1$, then the latter effect makes it socially optimal that each consumer consumes the products of A, that is $x' = 1$. Finally, the consumers' surplus is social welfare minus total profits.

Comparing social welfare in Γ^g and in Γ^s is simple if (α, k) belongs to Region Y in Figure 2. In this case in both games entry occurs and competition takes place under PB; we denote with $\bar{x}^{g*}$, $\bar{x}^{s*}$ the equilibrium average location of the marginal consumer in Γ^g and in Γ^s, respectively. Then, since $B1, B2$ are less aggressive than B (see Subsection 4.1.1), we have that $\bar{x}^{g*} < \bar{x}^{s*}$, and moreover $x' = 1$ since $\alpha > 1.415 > 1$ for each (α, k) in Region Y. Therefore in Γ^s firm A has a higher market share, which is welfare improving as $x' = 1$, that is $\bar{W}(\bar{x}^{g*}) < \bar{W}(\bar{x}^{s*})$. Regarding the consumers' surplus, firms make higher profits in Γ^s than in Γ^g and it turns out that the increase in total profits in moving from Γ^g to Γ^s is higher than the increase in social welfare. Hence, consumers' surplus is lower in Γ^s than in Γ^g, that is less fierce competition in Γ^s benefits firms and social welfare but harms consumers.

Regarding Region Z, it is useful to notice that given $\alpha \geq 1$, it is socially optimal that each consumer consumes products $A1, A2$, even if the consumer is located at $x_i = 1$ for $i = 1, 2$, because of the higher quality of products $A1, A2$. Therefore, social welfare is maximal when no rival of firm A enters, such that no entry cost is incurred and each consumer buys A's products. In Region Z, firm A is monopolist under Γ^g, that is B does not enter, and we know from Lemma 2 that A chooses PB with a bundle price slightly greater than $2v+2\alpha-2$. The price $2v+2\alpha-2$ would maximize social welfare as it would induce each consumer to buy products $A1, A2$, and a price slightly higher than $2v+2\alpha-2$ yields a social welfare close to the maximal value, higher than the social welfare in Γ^s, where $B1$ and $B2$ enter with positive probability. However, Γ^g leads to a much lower consumer surplus than Γ^s because under monopoly of A, consumers pay a high price for A's products. Therefore consumers are largely better off in Γ^s than in Γ^g.

Finally, in Region X the comparison between Γ^g and Γ^s is not as clear cut as in Regions Y and Z, because moving from IP under Γ^g to PB under Γ^s involves several effects. First, firm A obtains a greater market share because of the demand size effect generated by bundling and because firms $B1, B2$ are less aggressive than B; this implies $\alpha\bar{F}(\bar{x}^{s*}) > \alpha F(x^*)$. Second, $\bar{T}(\bar{x}^{s*}) > T(x^*)$ because under bundling consumers cannot mix and match products of different firms. This makes the comparison dependent on α and on k. For (α, k) in Region X below the dashed curve, the difference $\bar{T}(\bar{x}^{s*}) - T(x^*)$ turns out to be almost constant with respect to α, whereas $\alpha\bar{F}(\bar{x}^{s*}) - \alpha F(x^*)$ is increasing in α; this makes social welfare greater under Γ^s for $\alpha > 0.762$, whereas it is greater in Γ^g if $\alpha < 0.762$.[20] But as α increases, total profits increase more in Γ^s than in Γ^g, and consumers' surplus is lower (greater) in Γ^g for $\alpha < 0.997$ ($\alpha > 0.997$). For (α, k) in Region X above the dashed line, in Γ^s specialists enter according to (10), hence with positive probability A is monopolist in one or both markets; then entry cost are not incurred and (almost) all consumers buy only the products of firm A. Hence, welfare tends to be greater in Γ^s but consumers' surplus is lower because A's monopoly harms consumers.

Proposition 5 *Suppose that* $\alpha > 0.307$ *and* $k < \max\{\Pi^s_{B1}, \hat{\pi}\}$. *Then*
(i) for (α, k) *in Region* X *below the dashed curve, social welfare is higher in* Γ^g *(in* Γ^s*) for* $\alpha \in (0.307, 0.762)$ *(for* $\alpha \in (0.762, 1.415)$*) and consumers' surplus is higher in* Γ^s *(in* Γ^g*) for* $\alpha \in (0.307, 0.997)$ *(for* $\alpha \in (0.997, 1.415)$*);*
(ii) for (α, k) *in Region* Y*, social welfare is higher in* Γ^s*, consumers' surplus is higher in* Γ^g*;*
(iii) for (α, k) *in Region* Z*, social welfare is higher in* Γ^g*, consumers' surplus is higher in* Γ^s*.*

6 Conclusions

We have compared the effects of bundling in two settings in which a multiproduct dominant firm either faces an integrated rival or two separate rivals, and we have found that a key role

[20] This result, like the others in Proposition 5(i), is obtained using numeric analysis.

in this comparison is played by the level of dominance. For instance, for suitable values of the dominance level an integrated rival is more vulnerable to bundling than separate rivals. It would be interesting to study a more general model with $n \geq 2$ products, a dominant firm A offering products $A1, ..., An$, and n specialist firms $B1$,...,, Bn offering one product each, such that the latter firms may merge (perhaps partially) before deciding whether to enter or not. This may affect A's incentives to bundle, and the dominated firms' ability to withstand bundling.

References

[1] Armstrong, M. (1996), "Multiproduct Nonlinear Pricing", *Econometrica* 64(1), 51-75.

[2] Denicolo, V. (2000), "Compatibility and Bundling With Generalist and Specialist Firms", *Journal of Industrial Economics* 48(2), 177-188.

[3] Fang, H. and P. Norman (2006), "To Bundle or Not to Bundle", RAND Journal of Economics 37(4), 946-963.

[4] Hahn, J.-H. and S.-H. Kim (2012), "Mix-and-Match Compatibility in Asymmetric System Markets", Journal of Institutional and Theoretical Economics 168(2), 311-338.

[5] Hurkens, S., D.-S. Jeon and D. Menicucci (2018), "Dominance and Competitive Bundling", forthcoming in *American Economic Journal: Microeconomics.*

[6] Matutes, C. and P. Regibeau (1988), ""Mix and Match": Product Compatibility without Network Externalities", *RAND Journal of Economics* 19(2), 221-234.

[7] Nalebuff, B. (2000), "Competing Against Bundles", In *Incentives, Organization, Public Economics*, edited by Peter Hammond and Gareth D. Myles, London: Oxford University Press, 323-336.

[8] Nalebuff, B. (2004), "Bundling as an Entry Barrier", *Quarterly Journal of Economics* 119(1), 159-187.

[9] Peitz, M. (2008), "Bundling May Blockade Entry", *International Journal of Industrial Organization* 26(1), 41-58.

[10] Whinston, M. (1990), "Tying, Foreclosure and Exclusion", *American Economic Review* 80(4), 837-859.

7 Appendix

7.1 Proof of Lemma 2

In this proof it is useful to define $w = v + \alpha$.

(i) The profit function for A in market j is $p_{Aj}F(w - p_{Aj})$. We can restrict attention to $p_{Aj} \in [w-1, w]$ without loss of generality. Then $p_{Aj}F(w - p_{Aj}) = p_{Aj}(w - p_{Aj})$ and it is immediate to see that this function is maximized at $p_{Aj} = w - 1$ as long as $w \geq 2$.

(ii) The profit function for A is $p_A \bar{F}(w - p_A)$, and using (3) we find that it is maximized at $p_A = w - 1 + \frac{1}{2(w-1+\sqrt{(w-1)^2+\frac{3}{2}})}$, which is just slightly larger than $w - 1$ when w is large.

7.2 Proof of Lemma 5(ii-iii)

The proof of Lemma 5(ii) consists of four steps

Step 1 There exists no equilibrium such that $w - 1 < P_A - p_{B1} \leq w + 1$.

Proof of Step 1 If P_A, p_{B1} are such that $w - 1 < P_A - p_{B1} \leq w$, then $\Pi_A(P_A) = P_A \frac{1}{4}(2w + 2p_{B1} - 2P_A + 1)$, $\pi_{B1}(p_{B1}) = p_{B1}[1 - \frac{1}{4}(2w + 2p_{B1} - 2P_A + 1)]$ and the first order conditions are $\frac{1}{2}w + \frac{1}{2}p_{B1} + \frac{1}{4} - P_A = 0$ and $\frac{1}{2}P_A - \frac{1}{2}w + \frac{3}{4} - p_{B1} = 0$. They yield $P_A = \frac{1}{3}w + \frac{5}{6}$, $p_{B1} = \frac{7}{6} - \frac{1}{3}w$, but the inequality $w - 1 < P_A - p_{B1}$ is violated given that $w > 2$.

If P_A, p_{B1} are such that $w < P_A - p_{B1} \leq w + 1$, then $\Pi_A(P_A) = P_A \frac{1}{4}(1 + w + p_{B1} - P_A)^2$, $\pi_{B1}(p_{B1}) = p_{B1}[1 - \frac{1}{4}(1 + w + p_{B1} - P_A)^2]$ and the first order conditions are $\frac{1}{4}(1 + w + p_{B1} - P_A)(1 + w + p_{B1} - 3P_A) = 0$ and $-\frac{3}{4}p_{B1}^2 + (P_A - w - 1)\, p_{B1} + 1 - \frac{1}{4}\,(w - P_A + 1)^2 = 0$. They yield $P_A = \frac{1}{8}w + \frac{1}{8}\sqrt{w^2 + 2w + 17} + \frac{1}{8}$, $p_{B1} = \frac{3}{8}\sqrt{w^2 + 2w + 17} - \frac{5}{8}w - \frac{5}{8}$, but $p_{B1} > 0$ is violated given that $w > 2$.

Step 2 There exist P_A, p_{B1} such that $w - 2 \leq P_A - p_{B1} \leq w - 1$, denoted P_A^{s1}, p_{B1}^{s1}, which satisfy the first order conditions.

Proof of Step 2 If P_A, p_{B1} are such that $w - 2 \leq P_A - p_{B1} \leq w - 1$, then $\Pi_A(P_A) = P_A[1 - \frac{1}{4}(2 - w - p_{B1} + P_A)^2]$, $\pi_{B1}(p_{B1}) = p_{B1} \cdot \frac{1}{4}(2 - w - p_{B1} + P_A)^2$ and the first order conditions are $-\frac{3}{4}P_A^2 + (w + p_{B1} - 2)\, P_A + 1 - \frac{1}{4}\,(w + p_{B1} - 2)^2 = 0$, $\frac{1}{4}\,(2 - w + P_A - 3p_{B1})\,(2 - w + P_A - p_{B1}) = 0$. They yield $P_A^{s1} = \frac{5}{8}w + \frac{3}{8}\sqrt{w^2 - 4w + 20} - \frac{5}{4}$, $p_{B1}^{s1} = \frac{1}{8}\sqrt{w^2 - 4w + 20} - \frac{1}{8}w + \frac{1}{4}$, which satisfy the inequalities $w - 2 < P_A - p_{B1} < w - 1$, given that $w > 2$.

Step 3 Given that $p_{B1} = p_{B1}^{s1}$, for firm A it is optimal to play $P_A = P_A^{s1}$.

Proof of Step 3 For firm A, given that $p_{B1} = p_{B1}^{s1}$, the demand function is

$$D_A(P_A) = \begin{cases} 1 - \frac{1}{4}(2 - w - p_{B1}^{s1} + P_A)^2 & \text{if } p_{B1}^{s1} + w - 2 \leq P_A \leq p_{B1}^{s1} + w - 1 \\ \frac{1}{4}(2w + 2p_{B1}^{s1} - 2P_A + 1) & \text{if } p_{B1}^{s1} + w - 1 < P_A \leq p_{B1}^{s1} + w \\ \frac{1}{4}(1 + w + p_{B1}^{s1} - P_A)^2 & \text{if } p_{B1}^{s1} + w < P_A \leq p_{B1}^{s1} + w + 1 \end{cases}$$

and recall that P_A^{s1} belongs to the interval $[p_{B1}^{s1} + w - 2, p_{B1}^{s1} + w - 1]$. We first prove that D_A is a concave function of P_A for P_A in the interval $[p_{B1}^{s1} + w - 2, p_{B1}^{s1} + w]$ because $D_A'(P_A) =$

$-\frac{1}{2}(2-w-p_{B1}^{s1}+P_A)$ for $P_A \in [p_{B1}^{s1}+w-1, p_{B1}^{s1}+w]$, $D'_A(P_A) = -\frac{1}{2}$ for $P_A \in (p_{B1}^{s1}+w-1, p_{B1}^{s1}+w]$; hence $D'_A(P_A)$ is decreasing in the interval $[p_{B1}^{s1}+w-2, p_{B1}^{s1}+w]$. Since $D_A(P_A)$ is concave in $[p_{B1}^{s1}+w-2, p_{B1}^{s1}+w]$, it follows that also $\Pi_A(P_A) = P_A D_A(P_A)$ is concave in $[p_{B1}^{s1}+w-2, p_{B1}^{s1}+w]$, and therefore $P_A = P_A^{s1}$ is a best reply for firm A, relative to the interval $[p_{B1}^{s1}+w-2, p_{B1}^{s1}+w]$. For the interval $[p_{B1}^{s1}+w, p_{B1}^{s1}+w+1]$ it suffices to prove that Π_A is decreasing in P_A. We notice that $\Pi_A(P_A) = P_A\frac{1}{4}(1+w+p_{B1}^{s1}-P_A)^2$ and $\Pi'_A(P_A) = \frac{1}{4}(1+w+p_{B1}^{s1}-P_A)(1+w+p_{B1}^{s1}-3P_A)$; since $P_A > p_{B1}^{s1}+w$, it follows that $1+w+p_{B1}^{s1}-3P_A < 1-2w-2p_{B1}^{s1} < 0$.

Step 4 Given that $P_A = P_A^{s1}$, for firm $B1$ it is optimal to play $p_{B1} = p_{B1}^{s1}$.
Proof of Step 4 For firm $B1$, given that $P_A = P_A^{s1}$, the demand function is[21]

$$D_{B1}(p_{B1}) = \begin{cases} 1-\frac{1}{4}(2w+2p_{B1}-2P_A^{s1}+1) & \text{if } 0 \le p_{B1} < P_A^{s1}-w+1 \\ \frac{1}{4}(2-w-p_{B1}+P_A^{s1})^2 & \text{if } P_A^{s1}-w+1 < p_{B1} \le P_A^{s1}-w+2 \end{cases}$$

and $\pi_{B1}(p_{B1}) = p_{B1}D_{B1}(p_{B1})$ is the profit function of firm $B1$, with

$$\pi'_{B1}(p_{B1}) = \begin{cases} \frac{3}{4}+\frac{1}{2}P_A^{s1}-\frac{1}{2}w-p_{B1} & \text{if } 0 \le p_{B1} \le P_A^{s1}-w+1 \\ \frac{1}{4}\left(P_A^{s1}-w+2-p_{B1}\right)\left(P_A^{s1}-w+2-3p_{B1}\right) & \text{if } P_A^{s1}-w+1 < p_{B1} \le P_A^{s1}-w+2 \end{cases}$$

For $p_{B1} \in [0, P_A^{s1}-w+1]$, $\pi'_{B1}(p_{B1}) \ge \pi'_{B1}(P_A^{s1}-w+1) = \frac{1}{2}(w-P_A-\frac{1}{2}) > 0$. For $p_{B1} \in (P_A^{s1}-w+1, P_A^{s1}-w+2]$, $\pi'_{B1}(p_{B1})$ has the same sign as $P_A^{s1}-w+2-3p_{B1}$, which is positive for $p_{B1} < \frac{2}{3}-\frac{1}{3}w+\frac{1}{3}P_A^{s1}$, negative for $p_{B1} > \frac{2}{3}-\frac{1}{3}w+\frac{1}{3}P_A^{s1}$; thus π_{B1} is maximized at $p_{B1} = \frac{2}{3}-\frac{1}{3}w+\frac{1}{3}P_A^{s1}$, which coincides with p_{B1}^{s1}.

Proof of Lemma 5(iii). The equilibrium profit for A is $\Pi_A^{s1} = \left(1-\frac{1}{4}(2-w+\frac{3}{4}w-\frac{3}{2}+\frac{1}{4}\beta^{s1})^2\right)\left(\frac{5}{8}w+\frac{3}{8}\beta^{s1}-\frac{5}{4}\right)$, which is equal to $w-2+\frac{4w^2-16w+178}{(w^2-4w+40)\beta^{s1}+(w^2-4w+48)(w-2)}$, and it is routine to show that $\frac{4w^2-16w+178}{(w^2-4w+40)\beta^{s1}+(w^2-4w+48)(w-2)} < \frac{2}{w-2}$. From this inequality we infer that $\Pi_A^{s1} < v+2\alpha-2+\frac{2}{v+2\alpha-2}$, and (for $\alpha \in (0,3)$) $\pi_A^{s1} = \frac{(3+\alpha)^2}{18}+v+\alpha-1 > v+2\alpha-2+\frac{2}{v+2\alpha-2}$ is equivalent to $(3-\alpha)(9-\alpha) > \frac{36}{v+2\alpha-2}$, which holds as long as v is large enough.
For $\alpha > 3$, π_A^{s1} is equal to $v+2\alpha-2$, that is $w-2$, hence it is smaller than $\Pi_A^{s1} = w-2+\frac{4w^2-16w+178}{(w^2-4w+40)\beta^{s1}+(w^2-4w+48)(w-2)}$. The profit of B is $\frac{1}{4}(2-w+\frac{3}{4}w-\frac{3}{2}+\frac{1}{4}\beta^{s1})^2\left(\frac{1}{8}\beta^{s1}-\frac{1}{8}w+\frac{1}{4}\right) = \frac{2}{(w^2-4w+8)\beta^{s1}+(w-2)(w^2-4w+16)}$.

[21] There exists no p_{B1} such that $D_{B1}(p_{B1}) = 1-\frac{1}{4}(1+w+p_{B1}-P_A^{s1})^2$ because $P_A^{s1}-w<0$ for each $w>2$.