Dipartimento di Scienze Economiche
Università degli Studi di Firenze

Working Paper Series

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Working Paper N. 26/2012
November 2012

Dipartimento di Scienze Economiche, Università degli Studi di Firenze
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www.dse.unifi.it

Stampato in proprio in Firenze dal Dipartimento Scienze Economiche
(Via delle Pandette 9, 50127 Firenze) nel mese di Novembre 2012,
Esemplare Fuori Commercio Per il Deposito Legale
agli effetti della Legge 15 Aprile 2004, N.106

Cooperation and Endogenous Identity*

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October 18, 2012

Abstract

We consider individual identity as abstract common social kinship, and model it as the fuzzy degree of membership to sets of individuals. We connect identity to propensity of cooperation as modeled by a Prisoners' Dilemma game played in pairs of individuals in a mixed population of cooperators and defectors. Unlike in standard evolutionary game theory, individuals are identified with set dependant strategies; their fuzzy identity is adjusted in reaction to success/insuccess as measured by relative payoff. JEL classification number: C73, D01, Z13. Keywords: fuzzy identity, cooperation, evolutionary game theory.

1 Introduction and motivation

The notion of identity has recently become central for economic theory: see, e.g., [1].

Identity itself is a complex polysemic notion. Metaphysically, identity may be seen as a paradigmatic intrisic property ([4]); in the moral philosophy of decision theory, for example, it is related to the notion of responsability in Sen's consequentialism ([8]).

At another extreme, from the point of view of cultural anthropology, social philosophy, and again ethics, identity is sometimes viewed as a relation of the individual to a community (e.g., [11], [3]).

In this note, identity is construed as cultural (lingustic, religious, ethnic...) resemblance of an individual to others, or as partship in a community of kins. We employ this notion within a tradition of evolutionary game theory, in the context of the emergence of cooperation in pairwise interactions between individuals.

Identity is formalized here by the notion of membership of a set, and we make use of the abstractness of the notion ([2]). To illustrate, if the definition of being a Christian, as a member of the set of Christians, is not to be question begging, it will be necessary that the notion of a set of individuals is pre-assumed.

*Nicolò Bellanca and Pier Angiolo Cetica provided motivations to the paper. The usual disclaimer applies.

However, no set ontology has any realistic grounds: no natural matter, for example, may be appealed to in order to mark the difference between the President of Italy and the singleton set containing him as its sole member. Therefore, the notion of identity we appeal to is not simply not intrinsic to the individual, or to its environmental, or social, context; on top of that, it also concern a relation of the individual to something abstract, i.e., to sets, and it is to be interpreted better as a cognitive element regarding individuals.

An essential difference between evolutionary game theory ([9]), and game theory as a multi-person rational decision theory stands in the relation between players and their actions, or pure strategies. While in the latter traditional approach a player may implement one of a number of different actions, evolutionary game theory, in which usually players are living species, essentially identifies the species with some constant pattern of behaviour — or, more generically, a morphotype — which is, in principle, irrespective of circumstances. It will then be up to evolutionary forces to regulate the diffusion of morphotypes as a function of their success, and possibly drive them to extintion.

Models in evolutionary game theory have been used to study the emergence of cooperation in a society of individuals, where cooperation is an avantage to everyone, but where each single individual has a private incentive not to cooperate, whether or not the others do. Some of these models consider the possibility that the structure of pairwise encounters between individuals is in the form of a two persons Prisoners' Dilemma, but these encounters are constrained by sets: each individual is to encounter all and only the individuals belonging to the sets he also belongs to. Intuitively, if cooperators belong to sets where cooperators abound, and defectors meet prevalently defectors, then the former receive a larger average payoff, and cooperation may be overall more successful ([7], [5],[10], [6]).

The constraining set structure may receive alternative interpretations: ethologically, the set an individual belongs to may be a site of encounters, or an "ecological niche". Here, we wish to consider the anthropological reading of the set as a simple representation of the structure of reciprocal recognition between individuals, or the common "culture" of its members. Social identity of a single individual, then, is identified with membership to a set. Much in line with the set theoretical formalization of abstract properties, a religious, linguistic, or national identity of an individual is identified by his, or her, membership to communities of individuals with similar identities. Behaviour will be then aligned to abstract, or cognitive, elements, because it will be a function of sets; and our fuzzy sets will not vary as a function of degrees of membership of their elements.

On the other hand, one may consider that such identities are not always "crisp", and that the sense of belonging, say, to a religious community may come with a degree: in this paper, we formalize this by fuzzy degrees of membership to the relevant sets. Alternatively, the degrees of membership may be interpreted also as a "participation rate" in the site of encounters, by the individual.

2 The model

Consider a large population of individuals, each of them playing a Prisoners' Dilemma game (PD) with the others. For the sake of clarity in this exercise, consider the game to have cardinal payoffs, in the following simple specification. Each player may *Cooperate*, or *Defect.* The cooperator incurs a cost c, and benefits the opponent of b; the defector incurs no cost. Obviously, we assume $b > c$.

	C	D
C	$b-c, b-c$	$-c, b$
D	$b, -c$	$0, 0$

(1)

Given that D is a dominating action, if each individual plays the game with all others, then the subpopulation of individuals, i.e., the species playing D outfits the C species, and cooperation does not prevail.

Assume, for simplicity, that the population is organized into only two sets, A and B; let α_i the degree of membership of individual i to set A, and similarly for β_i with respect to set B. We assume, also out of simplicity, that $\alpha_i + \beta_i = 1$ for any one i; but the assumption may have some meaning under both intended interpretations, if the identities are somehow complementary, or the participation rates are constrained by a total amount of time.

We assume no structure on the identities/sets themselves; hence, the sets will not be closed under union or intersection, even though individuals may belong to more than one set, with varying degrees.

If $\alpha_i \geq 0.5$, i will be termed an α-individual; if $\beta_j \geq 0.5$, a β-individual.[1] We will characterize an α-individual with parameter α, and a β-individual with the corresponding parameter β: therefore, an α-individual i will belong to set A with degree α_i, and to set B with degree $(1-\alpha_i)$; a β-individual j will belong to A with degree $\left(1-\beta_j\right)$, and to B with degree β_j. Let π_α be the relative proportion of α-individuals, and let π_β be the complementary proportion of β-individuals.

It is standard in evolutionary game theory to identify species of individuals with pure strategies; but the set theoretical structure of encounters offers the flexibility to articulate behaviour with respect to "identities". Then, even if some individuals are characterized by an identity-independent behaviour, others will cooperate in one set, and defect in the other. It will be natural to assume that, in the latter case, cooperation is carried out in all, and only, encounters with individuals met in "one's own" identity set, whether or not the individual encountered is of one's own same identity.

Naturally, species are not usually, for example, predators (or preys) uniformely in all encounters; but game theoretical models may equally identify one species with one action, given that this may be

[1] If $\alpha_i = 0.5$, (or, equivalently, $\beta_i = 0.5$) i will be both an α-individual, and a β-individual. This ambiguity of the limiting case should cause no trouble.

interpreted asa predating, or as a predated, behaviour according to the different actions/species it is paired with.

As specified in the introduction, instead, here we take the view of identity as a social identification with a "community" of peers, and assume that the community, like sets, onthologically exists besides the individuals who, albeit only partially, belong to it. Hence, we may no longer use other species' actions to change the interpretation of anyone's, and we need to articulate a species in a variety of set dependant actions.

Hence, "separating" α-individual cooperate in A, and defect in B, and implement "set dependant" strategy CD; similarly, β-individual cooperate in B, and defect in A: the set dependant strategy DC. "Pooling" individuals would either always cooperate (set independant strategy CC), or always defect (set independant strategy DD); but pooling individuals may be assumed away at this stage.

Each encounter will give the payoffs of the game as in table 1, but each payoff will be multiplicatively weighted by the degrees of membership of the two players in the set "where" they meet. The weights are to be interpreted either as the "time" portion each individual spends in the site, or as an identity premium over the intrinsic payoffs of table 1. The total payoffs from encounters are assumed to be added up, as follows:

$$\begin{aligned} u_i^{CD} &= \alpha_i \left((b-c) \sum\nolimits_{j \neq i} \alpha_j - c \sum\nolimits_{k \neq i} (1-\beta_k) \right) + (1-\alpha_i)\, b \sum\nolimits_{\beta_k} \beta_k \\ u_i^{DC} &= (1-\beta_i)\, b \sum\nolimits_{j \neq i} \alpha_j + \beta_i \left((b-c) \sum\nolimits_{\beta_k} \beta_k - c \sum\nolimits_{j \neq i} (1-\alpha_j) \right) \end{aligned} \tag{2}$$

in an obvious notation.

Success of species will be measured, as it is common, by comparing the average payoff of each individual with overall average payoff of the whole population. We assume that only success of separating strategies will have a consequence on identity: presumably, individuals implementing pooling strategies are not sensitive to one's own identity. If the set dependant strategy CD of i is strictly successful, then α_i will increase; similarly, if the set dependant strategy DC of j is strictly successful, then β_j will increase. In the opposite cases, the corresponding degrees of identity move in the opposite direction. Hence, an equilibrium is reached whenever all average individual payoffs are equal, or in the boundary cases of identity degrees: 1 for successful individuals, and 0.5 for unsuccessful individuals.

To clarify, it is important to stress that, within the methodology of evolutionary game theory, individuals do not make strategic choices: hence, for example, an unsuccessful individual will not "switch" set dependant strategy and lower his, or her, membership degree below 0.5. It may be up to evolutionary forces, eventually, to drive to extinction individuals who remain strictly unsuccessful after identity adjustements.

From (2), given the existing proportion of species and given other individuals' fuzzy identities, any individual average payoff is linear in his/her own identity, as depicted in fig. 1.

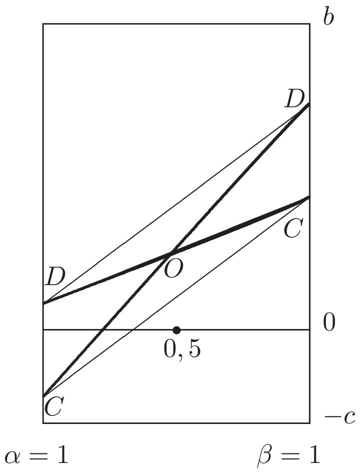


Figure 1:

The DD and CC segments are for reference only: they depict the corresponding strategies' average payoffs as a function of fuzzy membership, with α decreasing from 1 to 0, left to right, and, similarly β, from right to left. A set independant defector i receives an intrinsic payoff between b and 0, at each encounter. The extreme points of the DD segment stand for the average weighted payoffs when i belongs fully to set A, and set B, respectively. These depend on the individuals encountered in the sets, and they are also between b and 0. The CC segment is derived simply by shifting DD down by c. (The assumption of a large population will guarantee that the two individuals will have approximately the same encounters.)

Average payoffs of strategies CD and DC can be read as segments CO and OC, respectively. It can be shown that average payoffs $\bar{u}_i^{CD} = \bar{u}_j^{DC}$ when $\alpha_i = \beta_j = 0.5$.

The picture assumes that $\bar{u}_i^{DD}(\alpha_i = 1) < \bar{u}_j^{CC}(\beta_j = 1)$: a pooling cooperator fully in B will fare better than a pooling defector fully in A, the condition for success of cooperation discussed above. In our context, given that pooling individuals have zero weight, this requires that,

$$b\pi_\alpha \bar{\alpha}_{-i} < (b-c)\,\pi_\beta \bar{\beta}_{-i} - c\pi_\alpha\,(1-\bar{\alpha})$$

where $\bar{\alpha}$ stands for average identity of α-individuals, $\bar{\alpha}_{-i}$ stands for average identity of α-individuals other than i , and similarly for $\bar{\beta}_{-i}$; or, in a large population:

$$\frac{c}{b-c} < \bar{\beta}\frac{1-\pi_\alpha}{\pi_\alpha} - \bar{\alpha} \tag{3}$$

Condition (3) implies that DC increases scrictly with $\bar{\beta}$. Under this condition, if overall average $\bar{u}$ is above the middle point, then the identity of β-individuals is unstable at levels around $\bar{u}$, while α-individuals are all driven down to $\alpha = 0.5$. If $\bar{u}$ is below the middle point, then the identity of α-individuals is stable, and β-individuals are driven up to $\bar{\beta} = 1$.

3 Equilibria

Equilibria can be studied more easily via representative individuals of the species. Then, say, a single α-individual will be assigned weight π_α, and similarly for a single β-individual with weight π_β.

Whether or not, in equilibrium, all individuals of the same species (i.e., with the same set dependant strategy) have the same fazzy identity will depend on an analysis of stability, and it may be considered later.

Given that pooling species have zero weight, then $\pi_\alpha + \pi_\beta = 1$. Throughout the computation, we assume that $\pi_\alpha \leqslant 0.5$ (hence, $\pi_\beta \geqslant 0.5$): the opposite case is obtained obviously by swopping symbols uniformely.

The average payoffs of the two representative individuals, then, are

$$\begin{aligned} \bar{u}_\alpha &= (b-c)\,\alpha^2\pi_\alpha - c\,(1-\beta)\,\alpha\pi_\beta + b\,(1-\alpha)\,\beta\pi_\beta \\ \bar{u}_\beta &= (b-c)\,\beta^2\pi_\beta - c\,(1-\alpha)\,\beta\pi_\alpha + b\,(1-\beta)\,\alpha\pi_\alpha \end{aligned} \tag{4}$$

We wish to carry out the numerical exercise of finding equilibria. To this purpose, let $b = 9$, and $c = 3$. Then, equations (4) become

$$\begin{aligned} \bar{u}_\alpha &= 6\alpha^2\pi_\alpha - 3\,(1-\beta)\,\alpha\pi_\beta + 9\,(1-\alpha)\,\beta\pi_\beta \\ \bar{u}_\beta &= 6\beta^2\pi_\beta - 3\,(1-\alpha)\,\beta\pi_\alpha + 9\,(1-\beta)\,\alpha\pi_\alpha \end{aligned} \tag{5}$$

Equilibria can be computed by systems of "iso-identity" lines in the two-dimensional Cartesian space of average payoffs, together with the diagonal condition $\bar{u}_\alpha = \bar{u}_\beta$ for internal solutions of fuzzy identity steady states.

Let an iso-$\alpha(k,\pi)$ identity line be a function from $\bar{u}_\alpha$ to $\bar{u}_\beta$, with $\pi_\alpha = \pi$, and $\alpha = k$, such that $\bar{u}_{\alpha=k} = \bar{u}_\alpha(\beta, \alpha = k, \pi_\alpha = \pi)$, as in (5), and $\bar{u}_\beta\left(\beta = \bar{u}_\alpha^{-1}\left(\bar{u}_{\alpha=k}, \alpha = k, \pi_\alpha = \pi\right), \alpha = k, \pi_\alpha = \pi\right)$.

From the first of (5),

$$\beta = \frac{3k(1-\pi) - 6k^2\pi + \bar{u}_{\alpha=k}}{3k(1-\pi) + 9\,(1-k)\,(1-\pi)}$$

hence, by substitution into the second of (5),

$$\begin{aligned} \bar{u}_\beta &= 6\left(\frac{3k(1-\pi) - 6k^2\pi + \bar{u}_{\alpha=k}}{3k(1-\pi) + 9\,(1-k)\,(1-\pi)}\right)^2 (1-\pi) - 3\,(1-k)\left(\frac{3k(1-\pi) - 6k^2\pi + \bar{u}_{\alpha=k}}{3k(1-\pi) + 9\,(1-k)\,(1-\pi)}\right)\pi + \\ &\quad +9\left(1 - \left(\frac{3k(1-\pi) - 6k^2\pi + \bar{u}_{\alpha=k}}{3k(1-\pi) + 9\,(1-k)\,(1-\pi)}\right)\right)k\pi \end{aligned} \tag{6}$$

The construction of an iso-$\beta(k,\pi)$ identity line is similarly a function from $\bar{u}_\beta$ to $\bar{u}_\alpha$, with $\bar{u}_{\beta=k} = \bar{u}_\beta(\alpha, \beta = k, \pi_\alpha = \pi)$, and $\bar{u}_\alpha\left(\alpha = \bar{u}_\beta^{-1}\left(\bar{u}_{\beta=k}, \beta = k, \pi_\alpha = \pi\right), \beta = k, \pi_\alpha = \pi\right)$.

Again, from the second of (5),

$$\alpha = \frac{3k\pi - 6k^2(1-\pi) + \bar{u}_{\beta=k}}{3k\pi + 9\,(1-k)\,\pi}$$

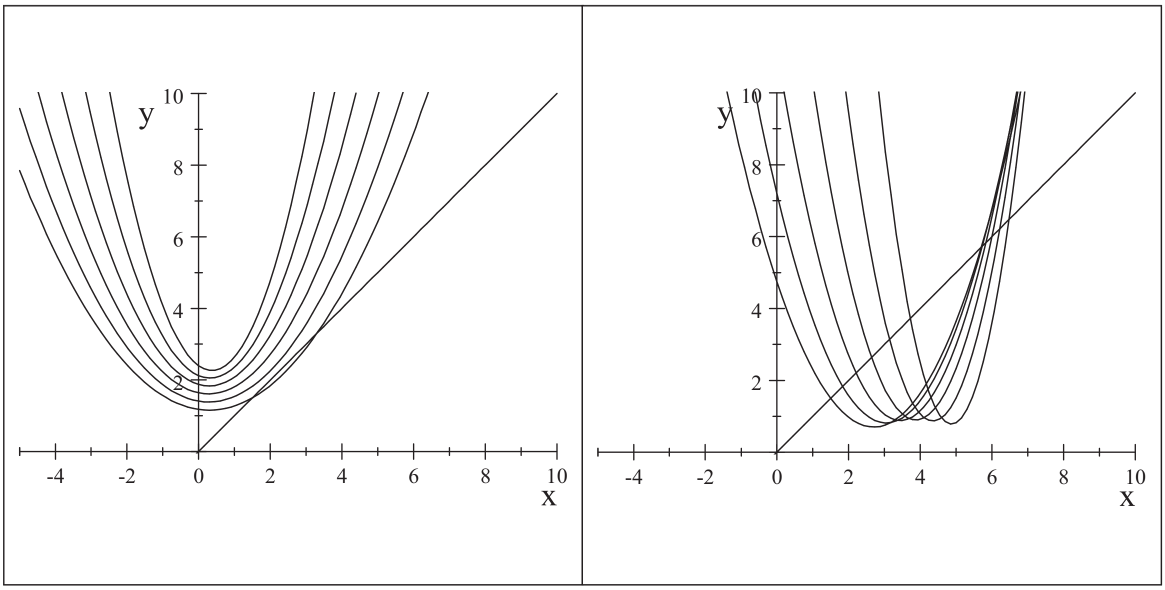
and

$$\begin{aligned} \bar{u}_\alpha &= 6\left(\frac{3k\pi - 6k^2(1-\pi) + \bar{u}_{\beta=k}}{3k\pi + 9\,(1-k)\,\pi}\right)^2 \pi - 3\,(1-k)\left(\frac{3k\pi - 6k^2(1-\pi) + \bar{u}_{\beta=k}}{3k\pi + 9\,(1-k)\,\pi}\right)(1-\pi) + \\ &\quad +9\left(1 - \left(\frac{3k\pi - 6k^2(1-\pi) + \bar{u}_{\beta=k}}{3k\pi + 9\,(1-k)\,\pi}\right)\right)k(1-\pi) \end{aligned} \tag{7}$$

Figure 8 shows on the left the map of iso-$\alpha(\pi_\alpha = 0.3, \alpha = k)$, for $k = 0.5$ to $k = 1$ in six equal steps: $k \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$. Here variable x stands for u_α, and variable y for u_β.

Iso-$\alpha(\pi_\alpha = 0.3, \alpha = 0.5)$ is the bottom line, while iso-$\alpha(\pi_\alpha = 0.3, \alpha = 1)$ is the top line.

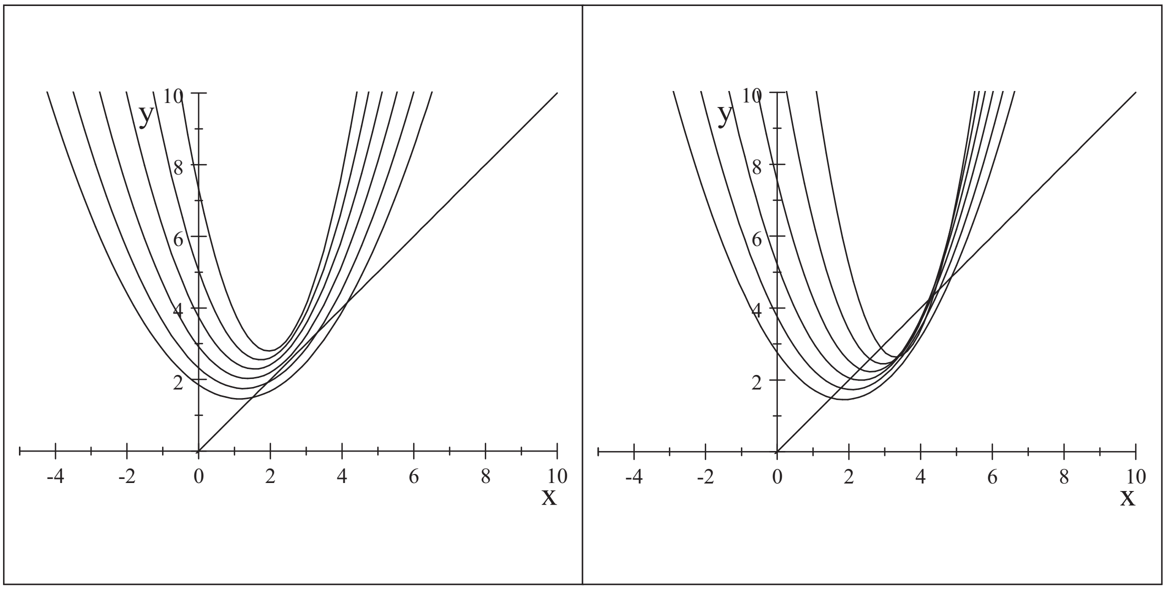
Correspondingly, we have on the right the map of iso-$\beta(\pi_\alpha = 0.3, \beta = k)$, for $k = 0.5$ to $k = 1$. Here variable x stands for u_β, and variable y for u_α.



iso-$\alpha(\pi_\alpha = 0.3, \alpha = k)$ iso-$\beta(\pi_\alpha = 0.3, \beta = k)$ (8)

Iso-$\beta(\pi_\alpha = 0.3, \alpha = 0.5)$ is the left most line, and iso-$\beta(\pi_\alpha = 0.3, \alpha = 1)$ is the righ tmost line.

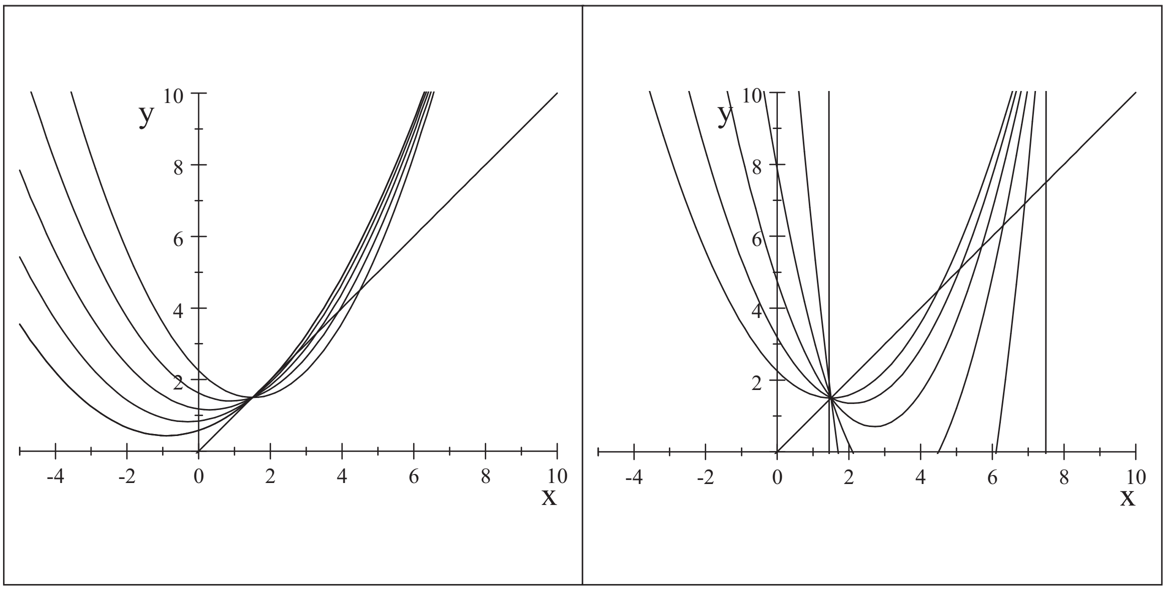
For further illustration, in figure 9 are the corresponding maps of iso-$\alpha(\pi_\alpha = 0.44, \alpha = k)$, and iso-$\beta(\pi_\alpha = 0.44, \alpha = k)$:



iso-$\alpha(\pi_\alpha = 0.44, \alpha = k)$ iso-$\beta(\pi_\alpha = 0.44, \beta = k)$ (9)

3.1 Internal equilibria

Point $(1.5, 1.5)$ of the diagonal is salient, for all iso-$\alpha(\pi_\alpha = \pi, \alpha = 0.5)$, and all iso-$\beta(\pi_\alpha = \pi, \beta = 0.5)$, $\pi > 0$, cross there, and it is therefore the inf of the half-open interval of egalitarian equilibria (a segment of the diagonal) whose length depends on π_α.[2] Figure 10 depicts the two parts of the statement, respectively. Again, x stands for $\bar{u}_\alpha$ on the left hand side, and for $\bar{u}_\beta$ on the right hand side; vice-versa for y.



iso-$\alpha(\pi_\alpha = \pi, \alpha = 0.5)$ iso-$\beta(\pi_\alpha = \pi, \beta = 0.5)$

(10)

For $\pi_\alpha = 0.3$, for example, the top point of the segment of equilibria solves $\bar{u}_\beta\,(\bar{u}_\alpha; \pi_\alpha = 0.3, \alpha = 0.5) = \bar{u}_\alpha$, with payoffs $(3.3, 3.3)$, and $\beta = 0.93$. At $\pi_\alpha = \frac{1}{3}$, the top equilibrium reaches payoffs $(3.5, 3.5)$, and $\beta = 1$.

The max of the equilibrium segment of equal payoffs is constrained by iso-$\alpha = 0.5$ up to $\pi = \frac{1}{3}$, and by iso-$\beta = 1$ afterwords. Hence, top equilibria for higher π_α must be computed along the iso-$\beta(\pi_\alpha = \pi, \beta = 1)$, by solving $\bar{u}_\alpha\,(\bar{u}_\beta; \pi_\alpha = \pi, \beta = 1) = \bar{u}_\alpha$. If $\pi = 0.4$, for example, the equilibrium payoffs are $(3.09, 3.09)$, with $\alpha = 0.57$; and if $\pi = 0.45$, the equilibrium payoffs are $(2.85, 2.85)$, with $\alpha = \frac{2}{3}$. Further increase of π_α brings the top equilibrium non-monotonically down to $(3.0, 3.0)$, for $\pi = 0.5$ and $\alpha = 1$.

Hence, $\pi_\alpha = \frac{1}{3}$ maximizes the top egualitarian equilibrium payoffs; here, identities are $\alpha = 0.5$ and $\beta = 1$, and payoffs $(3.5, 3.5)$ are achieved.

[2] If $\pi_\alpha = 0$, then iso-α lines are not defined, and equilibria are $\bar{u}_\beta = 6\beta^2$ for any β.

In order to spell out top equilibria at $\pi_\alpha \geq \frac{1}{3}$, take iso-$\beta(\pi_a = \pi, \beta = 1)$:

$$\begin{aligned}
\bar{u}_{\alpha=1} &= 6\left(\frac{3\,(1)\,\pi - 6\,(1)^2\,(1-\pi) + \bar{u}_{\beta=1}}{3\,(1)\,\pi + 9\,(1-(1))\,\pi}\right)^2 \pi + \\
&\quad -3\,(1-(1))\left(\frac{3\,(1)\,\pi - 6\,(1)^2\,(1-\pi) + \bar{u}_{\beta=1}}{3\,(1)\,\pi + 9\,(1-(1))\,\pi}\right)(1-\pi) + \\
&\quad +9\left(1-\left(\frac{3\,(1)\,\pi - 6\,(1)^2\,(1-\pi) + \bar{u}_{\beta=1}}{3\,(1)\,\pi + 9\,(1-(1))\,\pi}\right)\right)(1)\,(1-\pi)
\end{aligned}$$

Let payoffs be equal, and consider the explicit form of $\bar{u}$ as a function of π $(\pi \neq 0)$:

$$\bar{u} = -\frac{21}{2}\pi - \frac{3}{4}\sqrt{-20\pi + 4\pi^2 + 9} + \frac{33}{4} \tag{11}$$

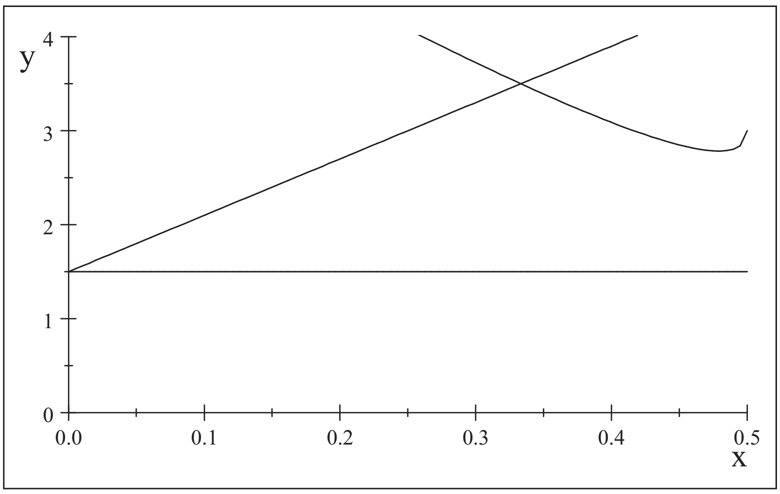
For $\pi_\alpha \leq \frac{1}{3}$, the max of equilibrium equal payoffs must be read on the iso-$\alpha\,(\pi_a = \pi, \alpha = 0.5)$.
Again, take iso-$\alpha(\pi_a = \pi, \alpha = 0.5)$, let payoffs be equal, and consider $\bar{u}$ as a function of π $(\pi \neq 0)$:

$$\begin{aligned}
\bar{u} &= 6\left(\frac{3\,(0.5)\,(1-\pi) - 6\,(0.5)^2\,\pi + \bar{u}}{3\,(0.5)\,(1-\pi) + 9\,(1-(0.5))\,(1-\pi)}\right)^2 (1-\pi) + \\
&\quad -3\,(1-(0.5))\left(\frac{3\,(0.5)\,(1-\pi) - 6\,(0.5)^2\,\pi + \bar{u}}{3\,(0.5)\,(1-\pi) + 9\,(1-(0.5))\,(1-\pi)}\right)\pi + \\
&\quad +9\left(1-\left(\frac{3\,(0.5)\,(1-\pi) - 6\,(0.5)^2\,\pi + \bar{u}}{3\,(0.5)\,(1-\pi) + 9\,(1-(0.5))\,(1-\pi)}\right)\right)(0.5)\,\pi
\end{aligned}$$

This solves as:

$$\bar{u} = 1.5 + 6\pi \tag{12}$$

Constraints (11) and (12) can now be plotted together in fig. 13:



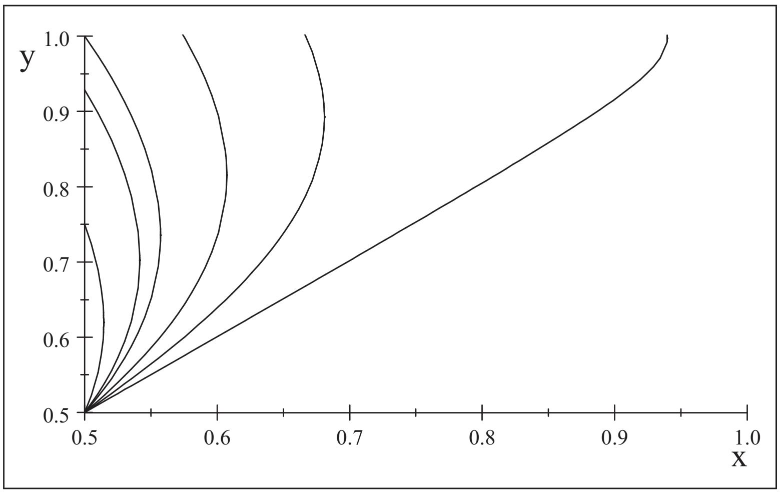
(13)

where x stands for π_α, and y stands for the common max equilibrium payoffs.

The equilibrium segments at varying π_α mark the vertical distance between the horizontal line at 1.5 and the min value of the two constraints.

Along the linear constraint, $\alpha = 0.5$, and β increases from 0.5 to 1. After $\pi = \frac{1}{3}$, $\beta = 1$, and α increases from 0.5 to 1.

One can trace the relationship between α and β along internal equilibria by equating $\bar{u}_\alpha = \bar{u}_\beta$ in (5), i.e., $6\alpha^2\pi_\alpha - 3(1-\beta)\alpha\pi_\beta + 9(1-\alpha)\beta\pi_\beta = 6\beta^2\pi_\beta - 3(1-\alpha)\beta\pi_\alpha + 9(1-\beta)\alpha\pi_\alpha$, and solving at different levels of $\pi_\alpha = 1 - \pi_\beta$, as in figure 11:



(14)

Here, x stands for α, and y for β. Levels of π_α are, from left to right: 0.2, 0.3, $\frac{1}{3}$, 0.4, 0.45, 0.499.

3.2 Boundary equilibria

If internal equilibria provide equal payoffs, boundary equilibria may be inegalitarian. On the other hand, these may be Pareto superior to egalitarian equilibria.

Boundary equilibria can be had with $\alpha = 0.5$, $\beta = 1$, and $\bar{u}_\alpha \leq \bar{u}_\beta$, because of the assumed dynamics of fuzzy identity. This implies that $\pi_\alpha \leq \frac{1}{3}$, for otherwise $\bar{u}_\alpha(\alpha = 0.5, \beta = 1) > \bar{u}_\beta(\alpha = 0.5, \beta = 1)$.

Boundary equilibria, for any $\pi_\alpha = \pi$, are solutions to the set of equations of iso-$\alpha = 0.5$ and iso-$\beta = 1$, respectively:

$$\begin{cases} \bar{u}_\beta = 6\left(\frac{3(0.5)(1-\pi)-6(0.5)^2\pi+\bar{u}_\alpha}{3(0.5)(1-\pi)+9(1-(0.5))(1-\pi)}\right)^2(1-\pi) - 3(1-(0.5))\left(\frac{3(0.5)(1-\pi)-6(0.5)^2\pi+\bar{u}_\alpha}{3(0.5)(1-\pi)+9(1-(0.5))(1-\pi)}\right)\pi + \\ \qquad +9\left(1-\left(\frac{3(0.5)(1-\pi)-6(0.5)^2\pi+\bar{u}_\alpha}{3(0.5)(1-\pi)+9(1-(0.5))(1-\pi)}\right)\right)(0.5)\pi \\ \bar{u}_\alpha = 6\left(\frac{3\pi-6(1-\pi)+\bar{u}_\beta}{3\pi}\right)^2\pi + 9\left(1-\left(\frac{3\pi-6(1-\pi)+\bar{u}_\beta}{3\pi}\right)\right)(1-\pi) \end{cases} \tag{15}$$

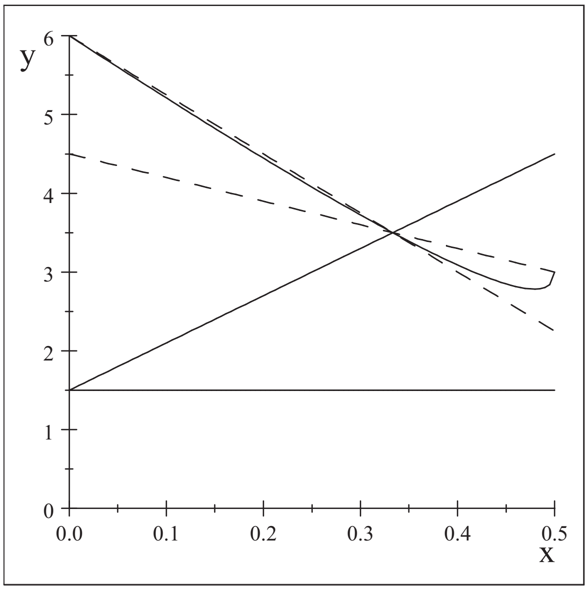
System (15) has four complex solutions; however, only one solution (the lowest real $\bar{u}_\beta$) respects both constraints on degrees of identity: $0.5 \leq \alpha \leq 1$, $0.5 \leq \beta \leq 1$. But one can compute that the explicit forms of average payoffs $\bar{u}_\alpha$, $\bar{u}_\beta$ as functions of π are in fact again simply linear: $\bar{u}_\beta = 6 - 7.5\pi$, and $\bar{u}_\alpha = 4.5 - 3.0\pi$, $0 < \pi \leq \frac{1}{3}$.

Numerical solutions of $\bar{u}_\alpha$, $\bar{u}_\beta$ at boundary equilibria are computed in table 16, and compared with values of $\bar{u}_\beta$ along the iso-$\beta = 1$ constraint for egalitarian equilibria, if this is extended to $\pi \leq \frac{1}{3}$.

π	$\bar{u}_\alpha$	$\bar{u}_\beta$	$\bar{u}_\beta$ at iso-β boundary for interior equilibria, when $\pi \leq \frac{1}{3}$
0.001	4.497	5.9925	5.992
0.01	4.47	5.925	5.9201
0.05	4.35	5.625	5.6024
0.1	4.2	5.25	5.21
0.2	3.9	4.5	4.4463
0.3	3.6	3.75	3.7252
$\frac{1}{3}$	3.5	3.5	3.5

(16)

Fig. 17 show how boundary solutions for $\pi \leqslant \frac{1}{3}$ plot; obviously, the upper dashed line is for $\bar{u}_\beta$, and the lower dashed line is for $\bar{u}_\alpha$ (dashed lines are to be read only up to $\pi = \frac{1}{3}$).



(17)

Internal and boundary equilibria

Pareto superiority of boundary equilibria, whenever possible, is evident. However, given the cardinal nature of the exercise, room is left for considerations of inequality aversions, which may alter the picture.

4 Stability

We have made only qualitative assumptions on the dynamics of degrees of identity with respect to success; therefore, we can only draw similarly qualitative conclusions on the stability of equilibria.

Consider the gradients of α and β around equilibria: if they are both convergent/divergent, then any composition of the two is also convergent/divergent. Otherwise, further postulations should be made on their relative weights in order to draw overall conclusions; in the absence of clear reasons, at this stage, we refrain from making special assumptions.

Non-egalitarian boundary equilibria are clearly locally stable, for they can only be perturbed with $\beta = 1 - \varepsilon$, and $\alpha = 0.5 + \delta$, $\varepsilon > 0$, $\delta > 0$; with $\bar{u}_\beta > \bar{u}_\alpha$ the assumptions on dynamics of α and β imply convergence to the equilibrium in both coordinates.

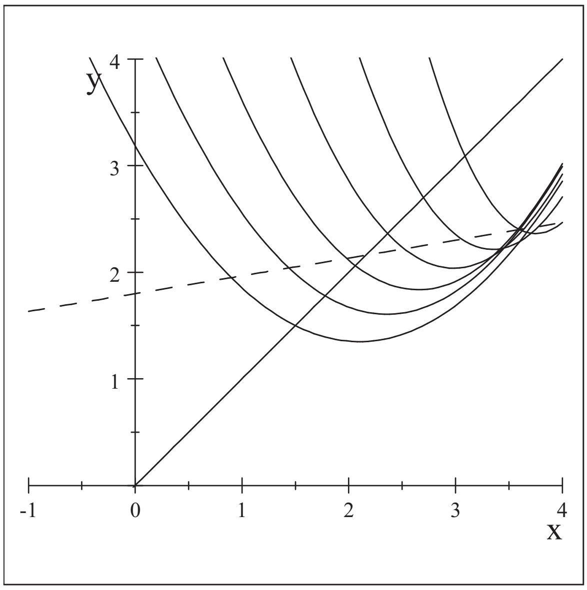
Egalitarian internal equilibria are always stable as far as α is concerned, on both sides, i.e., for $u_\beta < u_\alpha$ and $u_\beta < u_\alpha$; this is immediately seen on the maps of iso-α, for any $\pi_\alpha \leq 0.5$.

Identity β is divergent when (3) is satisfied, and it is convergent when the opposite hold. The threshold case is, in our example,

$$\frac{1}{2} = \beta\frac{1 - \pi_\alpha}{\pi_\alpha} - \alpha \tag{18}$$

As far as the iso-βs across the diagonal are concerned, they are locally stable on one side iff they are on the other as well, iff they are not orthogonal to the diagonal itself.

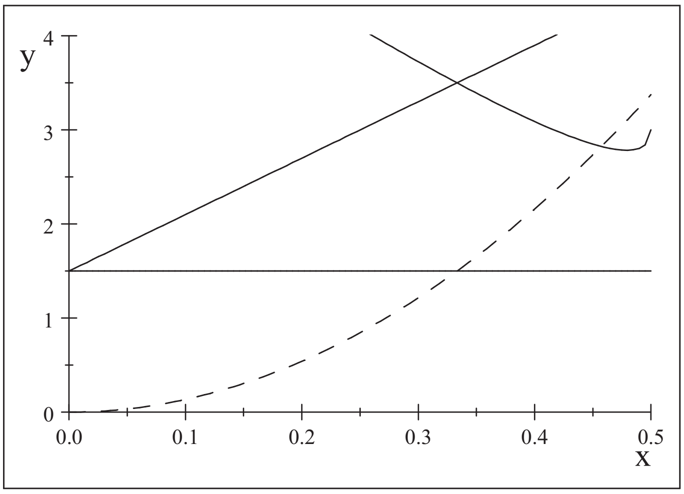
If $\pi_\alpha = 0.4$, for example, then (18) implies that $\beta = \frac{2}{3}\alpha + \frac{1}{3}$; when these two requirements are plugged into the (5), then $\bar{u}_\alpha = \frac{1}{6}\bar{u}_\beta + \frac{1}{8}$. This linear relation is plotted in fig. 19 as a dashed line, and it crosses the iso-βs where these have slope -1: it marks the upper boundary of the basin of attraction with respect to β.



(19)

It is for $\pi_\alpha = \frac{1}{3}$ that $\frac{d}{d\bar{u}_\alpha}\left(\text{iso-}\beta = 0.5, \pi_\alpha = \frac{1}{3}, (\bar{u}_\alpha = \bar{u}_\beta)\right) = -1$, i.e., at the point where iso-$\beta = 0.5$ crosses the diagonal. This is the lowest point of the lowest equilibrium where an iso-β have slope -1 at the point it crosses the diagonal. This threshold point moves upwards for higher π_α, up to where $\frac{d}{d\bar{u}_\alpha}(\text{iso-}\beta = 1, \pi_\alpha = \pi, (\bar{u}_\alpha = \bar{u}_\beta)) = -1$ at k just above 0.45.

In general, one can pair constraint (18) with the equilibrium equation between the two (5), and obtain that $\bar{u} = \frac{27}{2}\pi_\alpha^2$. This is depicted in fig. 20 as the dashed upper boundary of averall stable equilibria, for $\pi_\alpha \geqslant \frac{1}{3}$



(20)

5 Comments and conclusions

Equilibria we considered in this paper are reached via a dynamic which is not based on decision making payoff maximization motives. It is for this reason that the postulation for the dynamic which we suggested is exceedingly simple, and based on elementary common sense: in fact, adjustements in the degree of identity are the result of a process which may be imputed not solely to the individual, but also to his /her social environment. The degree of individual identity itself as a cognitive element may not be individualistic, but to be interpreted as a common “stigma”, or “pride”.

The model is, at best, a useful fragment of a more complex picture, where, for example, elements for incoming and outgoing processes are introduced. Natural selection of existing individuals may be called up, in order to discriminate between individuals with different degrees of success. For example, it is not obviuos that individuals *relatively* less successful should be driven to extention: unequal, but Pareto improving equilibria, may be a case in favour of absolute success. On the other hand, “immigration” (and “emigration”) should be part of the dynamic of the population, and this may reactivate a process of identity adjustments, which the calculated equilibria may have exhausted.

In the end, a full fledged dynamic study will be more interesting than the observation of equilibria, especially in view of the interplay between stable and unstable forces.

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