UNIVERSITÀ DEGLI STUDI FIRENZE

DISEI
DIPARTIMENTO DI SCIENZE PER L'ECONOMIA E L'IMPRESA

Working Papers - Economics

Technologies for Endogenous Growth

Federico Etro

Working Paper N. 20/2021

DISEI, Università degli Studi di Firenze
Via delle Pandette 9, 50127 Firenze (Italia) www.disei.unifi.it

Technologies for Endogenous Growth

by Federico Etro[1]

University of Florence

November, 2021

Abstract

I microfound endogenous growth through neoclassical technologies with substitutable inputs created by monopolistically competitive innovators. Investment delivers innovations of declining profitability, but increasing labor force generates growth depending on structural technological parameters that determine the elasticities of profits and output relative to the mass of inputs. With a Cobb-Douglas technology in labor and a CES aggregator of inputs growth declines with the substitutability between inputs, with a nested CES technology growth vanishes as long as the substitutability between labor and inputs is less than unitary, and with a Diewert technology growth is sustainable for a high share of inputs in production.

Key words: Long-run growth, Semi-endogenous growth, input substitutability, population, technology.

JEL Code: O3, O4.

[1]I am grateful to Paolo Bertoletti, Lilia Cavallari, Alessandro Cigno and other seminar participants for useful discussions. Nevertheless, any responsibility is mine. University of Florence, V. Pandette 32, Firenze, 50127. *federico.etro@unifi.it.*

Innovation-driven growth is the fruit of technology and appropriate market conditions. However, the relation between the aggregate production function and growth has been studied only under knife-edge assumptions. Endogenous growth models *à la* Romer (1990) and Barro and Sala-i-Martin (BS, 2004, Ch. 6) generate a balanced growth path under a Cobb-Douglas technology and a constant population level. Semi-endogenous growth models introduced by Jones (1995) and based on the same technological assumptions have avoided the scale effects in population by introducing "fishing-out" effects in the creation of new ideas: such externalities are crucial in determining the link between income growth and population growth in the long run also in more general models (Peretto, 1998; Cozzi, 2017). In this note I argue that a balanced growth path does not need to rely on knife-edge technological conditions, and it is the same nature of neoclassical technologies for the production of final goods that determines whether investment in the creation of new inputs by monopolistic providers can create sustainable growth.

I extend the canonical Barro and Sala-i-Martin model with population growth and a general neoclassical technology featuring substitutability between inputs, and I analyze monopolistic competition between their producers following recent advances on its foundations (Bertoletti and Etro, 2016, 2022; Parenti *et al.*, 2017; Etro, 2020). Substitutability between inputs, which is absent in standard models based on Romer (1987, 1990), implies that the marginal productivity of each new input decreases with the use of other inputs. Essentially, ideas are non-rival but may be substitutes: for instance, different dirty and clean technologies, different vaccines and treatments or different apps for cloud services are imperfect substitutes but they jointly contribute to increase the aggregate productivity. As a consequence, a given investment produces ideas that become gradually less profitable for a given market size, but an expanding market size can prevent the decline of growth. The mechanism and the long run growth rate depend structural parameters derived from the technological microfoundation.

To exemplify the nature of the results I start by using a basic Cobb-Douglas production function of labor and an aggregator of intermediate goods, where the latter satisfies Constant Elasticity of Substitution (CES),[2] nesting both the classic Romer (1990) technology with independent inputs and a neoclassical technology with perfectly substitutable intermediate inputs *à la* Solow (1956). Between these extreme cases, this specification allows for intermediate forms of imperfect substitutability, and I show that the long run growth of per capita income decreases in the same degree of substitutability between inputs and increases in their share in production: intuitively, the model generates a decreasing marginal profitability of innovation which reduces, but does not eliminate, the growth potential when population is expanding (unless the inputs are perfect substitutes and growth vanishes in the long run as in the Solow model).

This example can be extended to any more general technology satisfying constant returns to scale (CRS) in labor and inputs, providing either vanishing growth or a positive long run growth rate depending on two long run elasticities, namely those of profits and output with respect to the mass of inputs. Two additional unexplored examples illustrate the type of results that can emerge. In the case of a nested CES production function, which allows one to parametrize the elasticity of substitution between labor and a CES aggregator of intermediate goods, growth vanishes whenever this elasticity is less than unitary. Instead, in the case of a generalized linear technology (Diewert, 1971), long run growth is sustainable and depends on a structural parameter reflecting the share of inputs in production.

While I am not aware of microfoundations of semi-endogenous growth models through general technologies for final goods, some works have moved beyond standard microfoundations in a similar spirit (Bucci and Matveenko, 2017; Boucekkine *et al.*, 2017; Peretto, 2018; Latzer *et al.*, 2020). In particular,

[2] This is the technology suggested in BS (2004, Ch. 6) but, as far as I know, unexplored in the case of positive population growth.

Boucekkine *et al.* (2017) and Latzer *et al.* (2020) have recently studied endogenous growth models departing from the usual CES microfoundation of preferences and analyzing respectively indirectly and directly additive preferences, which generate variable markups. Contrary to these works, I focus on the microfoundation of technology, and show that demand systems for inputs generated by general aggregate production functions lead to novel results for the growth process. Related advances in the literature on macroeconomic models with endogenous entry and endogenous markups are discussed in Bilbiie *et al.* (2019), Cavallari (2013), Colciago and Rossi (2015), Dixon and Savagar (2020), Piersanti and Tirelli (2020) and elsewhere, though none of these works studies endogenous growth. In earlier work I have explored the role of technology in dynamic stochastic general equilibrium models with optimizing consumers (Etro, 2020), but with a focus on business cycle implications around a stationary equilibrium and without population growth.

In what follows Section 1 presents the general model discussing a basic example with a Cobb-Douglas production function and the case of a general CRS technology, Section 2 moves to a nested CES production function and Section 3 introduces a generalized linear production function. Finally, Section 4 concludes.

1 The model

Following the canonical Barro and Sala-i-Martin model with expanding variety of products (BS, 2004, Ch. 6), I consider a competitive final-good sector with a constant returns to scale technology in the labor force and a continuum of intermediate goods in $[0, N]$ produced by monopolistic innovators. Labor grows at a constant rate $n > 0$. The final good is the *numeraire* used for production of inputs at unitary cost, for the creation of new inputs at the fixed cost η for each innovaton, and for consumption. The economy is populated by identical and infinitely living households with utility:

$$U = \int_0^\infty e^{-\rho t} \log c dt \tag{1}$$

where $\rho > 0$ is the time preference rate and c individual consumption. This implies the standard Euler equation for individual consumption growth:

$$g = r - \rho$$

where r is the return rate. My focus will be on a balanced growth path in which the interest rate will be constant.

I consider a neoclassical production function satisfying CRS:

$$Y = F(\mathbf{X}, L)$$

where Y is output, L is the labor force, $\mathbf{X}$ is an infinite dimensional input profile in $[0, N]$ which is a *Lebsegue-measurable* mapping into $\mathbb{R}_+$, and F is assumed continuous, strictly quasi-concave and symmetric in $\mathbf{X}$. The intensive form for per capita output $y = Y/L$ can be defined on the basis of the per capita input $x_i = X_i/L$ as:

$$y = f(\mathbf{x}) \tag{2}$$

where $f(\mathbf{x}) \equiv F(\mathbf{X}/L, 1)$ by CRS. As long as the marginal productivity of an input depends on its quantity only, as in Romer (1990), or on an aggregator of all the inputs that is taken as given under monopolistic competition, we can easily compute the pricing rules and, through that the profits of the firms and the equilibrium production and growth. I will start by presenting a simple example of this case based on a Cobb-Douglas technology in labor and a CES aggregator of inputs, and then I will consider more general technologies.

1.1 A Cobb-Douglas example

Consider the following Cobb-Douglas production function in extensive form:

$$Y = \left(\int_0^N X_j^\sigma dj \right)^{\frac{\alpha}{\sigma}} L^{1-\alpha}$$

where $\alpha \in (0,1)$ reflects the relative importance of inputs in production and $\sigma \in (\alpha, 1)$ reflects their substitutability. BS (2004, Ch. 6, Problem 6.2) have considered this specification without exploring the extension with population growth, whose implications have not been noticed as far as I know. The extreme case with $\sigma = \alpha$ brings to the production function $Y = \int_0^N X_j^\alpha L^{1-\alpha} dj$ used by Romer (1990), BS (2004) and at the basis of most of the endogenous growth literature, where the inputs (and their demands) are independent between each other. Instead, the other extreme case $\sigma = 1$ brings to a standard neoclassical production function where the inputs are perfectly substitutable and what matters is their total amount $X = \int_0^N X_j dj$ in a production function as $Y = X^\alpha L^{1-\alpha}$. Intermediate cases are my main interest.

The intensive form reads as:

$$y = \left(\int_0^N x_j^\sigma dj \right)^{\frac{\alpha}{\sigma}} \tag{3}$$

With this technology the inverse demand of input i is:

$$p_i = \frac{\alpha x_i^{\sigma - 1}}{\left(\int x_j^\sigma dj \right)^{1 - \frac{\alpha}{\sigma}}}$$

Under monopolistic competition each input producer sets either prices or quantities to maximize profits $\pi_i (p_i - 1) x_i L$ taking as given the aggregator. This delivers the constant price:

$$p = \frac{1}{\sigma}$$

allowing us to solve for the equilibrium production of each firm as:

$$x(N) = \left(\alpha \sigma N^{\frac{\alpha - \sigma}{\sigma}} \right)^{\frac{1}{1-\alpha}}$$

which declines in the mass of inputs N under our assumption $0 < \alpha < \sigma < 1$. Accordingly, the symmetric profits of each producer are:

$$\pi(N) = \left(\frac{1-\sigma}{\sigma} \right) \left(\alpha \sigma N^{\frac{\alpha - \sigma}{\sigma}} \right)^{\frac{1}{1-\alpha}} L \tag{4}$$

and they also decrease when the mass of inputs used in production increases (while they are constant in standard endogenous growth models *à la* Romer). Since all producers are identical, these are also the profits of the marginal producer deciding whether to enter or not. The property of decreasing marginal profitability is what limits growth in the long run, as the property of decreasing marginal productivity of capital is what limits growth in neoclassical models of capital accumulation.

The free entry condition:

$$\frac{\pi(N)}{r} = \eta \tag{5}$$

implies that the interest rate can be constant in the long run (to insure constant consumption growth through the Euler equation) if and only if the growth rate of the mass of intermediate goods satisfies:

$$\frac{\dot{N}}{N} = \frac{\sigma(1-\alpha)n}{\sigma-\alpha} \tag{6}$$

Since income per capita in a symmetric equilibrium must be:

$$\begin{aligned} y(N) &= N^{\frac{\alpha}{\sigma}} x(N)^{\alpha} = \\ &= (\alpha\sigma)^{\frac{\alpha}{1-\alpha}} N^{\frac{\alpha}{\sigma}+\frac{\alpha(\alpha-\sigma)}{\sigma(1-\alpha)}} \end{aligned} \tag{7}$$

after using the equilibrium production of the inputs, the long run growth rate can be derived from (6) as follows:

$$g = \frac{\alpha(1-\sigma)n}{\sigma-\alpha} \tag{8}$$

As in semi-endogenous growth models, this rate is linear in the population growth rate. What is new is that the growth rate increases in α, which represents the share of intermediate goods in production, and decreases in σ, due to a lower marginal profitability of innovation (and therefore lower investments). As well known, in the presence of population growth, per capita income growth becomes explosive in the Romer case ($\sigma = \alpha$) and remains null in the neoclassical Solow case ($\sigma = 1$). Intuitively, investment generates sustainable growth when it creates new inputs that are not identical to the existing ones (as, instead, they are in the neoclassical case of new capital), but are imperfect substitutes of the existing inputs, and therefore enrich the technological frontier. As I will show next, similar implications can apply beyond this example.

1.2 General aggregate production functions

I now provide a more general characterization of the long run growth rate based on recent advances in the theory of monopolistic competition (see Bertoletti and Etro, 2016, 2017; d'Aspremont and Dos Santos Ferreira, 2016; Parenti *et al.*, 2017; Bertoletti *et al.*, 2018). Few technical assumptions are needed to deal with general CRS technologies with a continuum of intermediate goods. In particular, following Parenti *et al.* (2017, pp. 92-93), I assume that the functional f is *Fréchet differentiable* in $\mathbf{x} \in L_2$, in the sense that there exists a unique function $D(x_i, \mathbf{x})$ such that, for all $\mathbf{h} \in L_2$, the equality:

$$f(\mathbf{x}+\mathbf{h}) = f(\mathbf{x}) + \int_0^N D(x_i, \mathbf{x}) h_i di + O\left(\|\mathbf{h}\|_2\right)$$

holds, where $\|.\|_2$ is the L_2-norm (Dunford and Schwartz, 1988) and $D(x_i, \mathbf{x})$ is the marginal productivity of input i, which is assumed twice continuously differentiable in x_i with $\partial D(x_i, \mathbf{x})/\partial x_i < 0$. Denoting the symmetric per capita output $\tilde{f}(x, N) \equiv f(x\mathbf{I}_N)$ where $\mathbf{I}_N$ is the indicator of $N \sqsubseteq \mathbb{R}_+$, I assume this to be continuous and differentiable in N with $\partial \tilde{f}(x, N)/\partial N > 0$ for any x.

The inverse demand of input i is:

$$p_i = D(x_i, \mathbf{x})$$

Since $\mathbf{x}$ is defined up to a zero measure set, the cross elasticity is null, but substitutability between inputs holds when an increase of the mass of inputs used in positive quantity reduces the marginal productivity and the inverse demand of the other inputs. Denoting the inverse demand under symmetry as

$\tilde{D}(x,N) \equiv D(x, x\mathbf{I}_N)$, I assume this to be continuous and differentiable in N with $\tilde{D}_N(x,N) < 0$ for any x.

Monopolistic competition generates prices:

$$p = \frac{\varepsilon(x,N)}{\varepsilon(x,N)-1} \quad \text{with } \varepsilon(x,N) \equiv \frac{-D(x,x\mathbf{I}_N)}{xD'(x,x\mathbf{I}_N)} \tag{9}$$

where the symmetric elasticity of substitution for the mass of inputs (due to the symmetry of the production function) is assumed to satisfy $\varepsilon(x,N) > 1$ (see Parenti *et al.*, 2017, pp. 95-6 for the related conditions). This elasticity is actually a constant in standard models (including our earlier example based on a Cobb-Douglas technology) and in the example of Section 2 based on a nested CES technology. However, in Section 3 I will present an example based on a generalized linear technology where this elasticity is decreasing in the mass of inputs and approaches a constant only in the long run. Such asymptotic convergence to a finite constant, namely $\varepsilon(x,\infty) \to \varepsilon \in (1,\infty)$, is the assumption I will adopt in general: this is necessary for a balanced growth path because it insures a constant positive markup in the long run, which is needed to feed profitability of investment in the creation of new inputs.[3]

Given the pricing rule, the production of each input in the long run is defined by the market clearing condition:

$$\tilde{D}(x,N) = \frac{\varepsilon}{\varepsilon - 1}$$

Since the left hand side decreases in both x and N (due to concavity of the production function and the assumption on substitutability of inputs), the symmetric equilibrium quantity $x(N)$ declines in the mass of inputs. Its elasticity is defined in absolute value by the function $\beta(N) = -\frac{d\ln x}{d\ln N}$, and a balanced growth path with positive population growth can only exist if $\beta(\infty) \to \beta \in (0,\infty)$, which is the natural assumption I will make in what follows. Notice that the

[3] Translog technologies lead to elasticities of substitution that increase indefinitely with the mass of inputs (see Bertoletti and Etro, 2016) which makes long run growth unsustainable.

profit elasticity is null in the Romer model, and it was a constant in the earlier example based on a Cobb-Douglas example. However, in both the examples of Section 2 based on a nexted CES technology and Section 3 based on a generalized linear technology, the profit elasticity $\beta(N)$ will change with the mass of inputs and approach a constant only in the long run.

Given the equilibrium production of each monopolist, its long run profits become:

$$\pi(N) = \frac{x(N)L}{\varepsilon - 1}$$

with the same long run elasticity β, and the free entry condition $\pi(N) = \eta r$ implies that the interest rate can be constant in the long run if and only if $\beta(\dot{N}/N) = n$ implying:

$$\frac{\dot{N}}{N} = \frac{n}{\beta} \tag{10}$$

Finally, equilibrium output is:

$$y(N) = \tilde{f}(x(N), N)$$

and its elasticity with respect to the mass of inputs is defined as $\vartheta(N)$ with $\vartheta(\infty) \to \vartheta \geqslant 0$. Clearly, if $\vartheta = 0$ growth cannot be sustainable in the long run, a case exemplified in Section 2 under a nested CES technology for some parameter values. Instead, if $\vartheta > 0$ due to relevant gains from variety in the production function, growth is possible: this case will emerge in Section 3 under a generalized linear technology, when the output elasticity will depend on the mass of inputs and approach a positive constant in the long run.

When $\vartheta > 0$ the growth rate of income is:

$$g = \vartheta \frac{\dot{N}}{N}$$

and, using (10), it approaches a positive constant:

$$g = \frac{\vartheta n}{\beta} \tag{11}$$

which depends on population growth through the two structural long run elasticities. To close the model, the Euler condition and the resource constraint deliver the equilibrium interest rate and per capita consumption.

In practice, economic growth is driven by population growth through the long run elasticity β, which represents the extent to which new ideas become less profitable (or harder to get) when there are more of them, and the long run elasticity ϑ, which represents the degree of gains from variety in production (or of increasing returns to scale in ideas). They depend on technological conditions. For instance, in the Cobb-Douglas example the elasticity of profits (4) and output (7) are:

$$\beta = \frac{\sigma - \alpha}{\sigma(1-\alpha)} \quad \text{and} \quad \vartheta = \frac{\alpha(1-\sigma)}{\sigma(1-\alpha)}$$

providing the growth rate (8) in function of the structural parameters α and σ.

2 A nested CES production function

Constant long run growth can be a knife-edge outcome depending on precise technological conditions. To see why this is the case also in our environment, I now generalize further the Cobb-Douglas example moving to a nested CES technology (*à la* Arrow *et al.* 1961) in labor and a CES aggregator of the intermediate inputs. As before, this delivers an elasticity of substitution between inputs which is perceived as constant by monopolistically competitive producers, but it also deliver elasticities of profits and output that depend on the mass of inputs. Their long run behavior determines the growth potential.

In particular, let us consider a nested CES production function represented in extensive form as follows:

$$Y = \left[(1-\alpha)L^{-\psi} + \alpha H^{-\psi}\right]^{-\frac{1}{\psi}} \quad \text{with } H \equiv \left(\int_0^N X_j^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}}$$

The parameter $\psi > -1$ is inversely related to the elasticity of substitution between labor and intermediate goods, with a Cobb-Douglas technology in the limit case where $\psi \to 0$, a Leontief technology in the limit where $\psi \to \infty$, and perfect sustitutability between labor and inputs for $\psi \to -1$. The parameter $\alpha \in (0,1)$ determines always the shares of intermediate goods and labor in total production. Finally, $\theta > 1$ parametrizes the elasticity of substitution between varieties of intermediate goods. The intensive form reads as follows:

$$y = \left[1 - \alpha + \alpha \left(\int_0^N x_j^{\frac{\theta-1}{\theta}} dj\right)^{\frac{-\theta\psi}{\theta-1}}\right]^{-\frac{1}{\psi}} \tag{12}$$

In this case the inverse demand of inputs is:

$$p_i = \alpha x_i^{-\frac{1}{\theta}} \left[1 - \alpha + \alpha \left(\int_0^N x_j^{\frac{\theta-1}{\theta}} dj\right)^{\frac{-\theta\psi}{\theta-1}}\right]^{-\frac{1}{\psi}-1} \left(\int_0^N x_j^{\frac{\theta-1}{\theta}} dj\right)^{\frac{-\theta\psi}{\theta-1}-1}$$

and the monopolistic competition price (maximizing profits taking as given the aggregator of intermediate inputs) can be derived as:

$$p = \frac{\theta}{\theta - 1}$$

Given this, the symmetric equilibrium production of inputs can be computed as follows:

$$x(N) = \left(\frac{\alpha(\theta-1)}{\theta}\right)^{\frac{1}{1+\psi}} N^{\frac{-(\theta\psi+\theta-1)}{(1+\psi)(\theta-1)}} \left[\frac{1 - \alpha^{\frac{1}{1+\psi}} \left(\frac{\theta}{\theta-1}\right)^{\frac{\psi}{1+\psi}} N^{\frac{-\psi}{(1+\psi)(\theta-1)}}}{1-\alpha}\right]^{\frac{1}{\psi}}$$

which is always decreasing in the mass of firms when $\psi \geqslant 0$. Its elasticity can be computed as:

$$\beta(N) = \frac{\theta - 1 + \theta\psi - \left[\alpha^{\frac{-1}{1+\psi}} \left(\frac{\theta}{\theta-1}\right)^{\frac{\psi}{1+\psi}} N^{\frac{\psi}{(1+\psi)(\theta-1)}} - 1\right]^{-1}}{(1+\psi)(\theta-1)}$$

which converges to a constant. However, the equilibrium per capita income is:

$$y(N) = \left[1 - \alpha + \alpha N^{\frac{-\theta\psi}{\theta-1}} x(N)^{-\psi}\right]^{-\frac{1}{\psi}}$$

whose elasticity can be computed as:

$$\vartheta(N) = \frac{\alpha^{\frac{1}{1+\psi}} \left[\frac{\theta}{\theta-1} - \beta(N)\right]}{\left(\frac{\theta-1}{\theta}\right)^{\frac{\psi}{1+\psi}} N^{\frac{\psi}{(1+\psi)(\theta-1)}}}$$

and this elasticity converges to zero for any $\psi > 0$, delivering zero long run growth in spite of increasing population. Instead, when labor and intermediate goods are highly substitutable ($\psi < 0$) growth tends to explode and the model lacks a balanced growth path. Long run growth is sustainable only in the Cobb-Douglas case, which is consistent with our initial example. To verify this, notice that the limit case of $\psi \to 0$ implies the elasticities:

$$\beta = \frac{\theta - 1 - \frac{\alpha}{1-\alpha}}{\theta - 1} \quad \text{and} \quad \vartheta = \alpha \left(\frac{\theta}{\theta - 1} - \beta \right)$$

delivering the growth rate:

$$g = \frac{\alpha n}{\theta(1-\alpha) - 1}$$

as long as this is positive, which requires high substitutability between intermediate goods.[4]

3 A Diewert production function

I finally present an example of a different CRS technology due to Diewert (1971) that generates positive long run growth. As far as I am aware, this type of technology has not been used in endogenous growth investigations.[5] It delivers an elasticity of substitution between intermediate inputs that is not constant, but decreases in the mass of inputs. Nevertheless this elasticity and therefore the monopolistic competition prices converge to a constant, as do the other relevant elasticities, leading to a balanced path of endogenous growth.

Consider the generalized linear production function:

$$Y = \left[\int_0^N \left(\gamma X_i + \int_0^N \sqrt{X_i X_j} dj \right) di \right]^{\alpha} L^{1-\alpha}$$

[4] The growth rate corresponds to the one for the Cobb-Douglas example after converting the parameters with $\sigma = 1 - 1/\theta$.

[5] Applications of a more general Qmor specification for preferences are available in trade (Feenstra, 2018).

with $\alpha < 1/2$ reflecting as usual the share of intermediate goods in total production and $\gamma > 0$ parametrizing substitutability between these inputs. In intensive form, this simplifies to:

$$y = \left[\int_0^N \left(\gamma x_i + \int_0^N \sqrt{x_i x_j} dj \right) di \right]^\alpha \tag{13}$$

The inverse demand of inputs is:

$$p_i = \frac{\alpha \left(\gamma + \frac{1}{2\sqrt{x_i}} \int_0^N \sqrt{x_j} dj \right)}{\left[\gamma \int_0^N x_j dj + \int_0^N \int_0^N \sqrt{x_k x_j} dj dk \right]^{1-\alpha}}$$

Under monopolistic competition each input provider i selects the quantity of input x_i taking as given the aggregators that affect demand. This provides the relevant elasticity $\varepsilon(x, N) = 2 + \frac{4\gamma}{N}$ which converges to a constant for $N \to \infty$. The monopolistic price converges therefore to $p = 2$ in the long run. The equilibrium production of inputs can be computed as:

$$x(N) = \left[\frac{\alpha(2\gamma + N)}{4(\gamma + N)} N^{1-2(1-\alpha)} \right]^{\frac{1}{1-\alpha}}$$

with elasticity:

$$\beta(N) = 2 - \frac{1 - \frac{\gamma N}{(2\gamma + N)(\gamma + N)}}{1 - \alpha}$$

which clearly converges to $\beta = 2 - \frac{1}{1-\alpha}$ in the long run and is positive under our assumptions.

The equilibrium income in per capita terms is:

$$y(N) = [N(\gamma + N)x(N)]^\alpha$$

whose elasticity is instead:

$$\vartheta(N) = \alpha \left(2 - \frac{\gamma}{\gamma + N} - \beta(N) \right)$$

and converges to $\vartheta = \frac{\alpha}{1-\alpha}$. Then, the long run growth rate can be derived as:

$$g = \frac{\alpha n}{2(1-\alpha) - 1} \tag{14}$$

Since the substitutability between intermediate inputs and between labor and intermediate inputs approach asymptotically finite numerical values, the only relevant technological parameter is the one reflecting the share of inputs in production, and growth increases in α due to higher importance of the intermediate goods in production.

4 Conclusion

Technological conditions shape the relation between population growth and income growth in models of endogenous technological progress. While this relation has been implicit in semi-endogenous growth models based on externalities induced by the mass of firms on aggregate productivity (Jones, 1995), it can be made explicit under general neoclassical CRS production functions. What is needed is to abandon the independence between accumulating inputs assumed by Romer (1990) and BS (2004) and inherited by most of the subsequent endogenous growth literature, and to take into account input substitutability.

Further avenues for future research appear interesting. First, one may want to consider the welfare implications of long run growth. Notice that the conventional assumption of symmetric production functions implies that the order in which new inputs are created and adopted is irrelevant.[6] But in practice different inputs contribute differently to aggregate production, and the choice of the order in which new inputs are created by the market is crucial to explain economic development. Such a choice depends on the profitability of inputs and not on their contribution to future innovations, and can be in conflict with the choice that a social planner would make in the interest of consumers: further investigations of endogenous product introduction under asymmetric technologies could be useful.[7] Second, the current model considered population growth as

[6] The same applies to Schumpeterian models of growth (Aghion and Howitt, 1992), where there is a natural order in which new (better) inputs are created.

[7] A preliminary exploration is in Bertoletti and Etro (2022).

an exogenous determinant of the rates of technological progress and growth, but it would be interesting to analyze the opposite channel where economic growth affects fertility. Finally, the microfoundation based on more general aggregate production functions, and possibly some of the functional forms presented here, could be used for quantitative evaluations of endogenous growth models.

References

Aghion, Philippe and Peter Howitt, 1992, A Model of Growth through Creative Destruction, *Econometrica*, 60, 323-351.

d'Aspremont, Claude and Rodolphe Dos Santos Ferreira, 2016, Oligopolistic vs. Monopolistic Competition: Do intersectoral effects matter?, *Economic Theory*, 62, 1, 299-324.

Arrow, Kenneth, Hollis Chenery, Bagicha Minhas and Robert Solow, 1961, Capital-labor Substitution and Economic Efficiency, *Review of Economics and Statistics*, 43, 225-250.

Barro, Robert and Xavier Sala-i-Martin, 2004, *Economic Growth*, MIT Press, Cambridge.

Bertoletti, Paolo and Federico Etro, 2016, Preferences, Entry and Market Structure, *RAND Journal of Economics*, 47, 4, 792-821.

Bertoletti, Paolo and Federico Etro, 2017, Monopolistic Competition when Income Matters, *Economic Journal*, 127, 603, 1217-1243.

Bertoletti, Paolo and Federico Etro, 2022, Monopolistic Competition, as you like it, *Economic Inquiry*, 60, 1, 1-27.

Bertoletti, Paolo, Federico Etro and Ina Simonovska, 2018, International Trade with Indirect Additivity, *American Economic Journal: Microeconomics*, 10, 2, 1-57.

Bilbiie, Florin, Fabio Ghironi and Marc Melitz, 2019, Monopoly Power and Endogenous Product Variety: Distortions and Remedies, *American Economic Journal: Macroeconomics*, 11, 4, 140-74.

Boucekkine, Raouf, Hélène Latzer and Mathieu Parenti, 2017, Variable Markups in the Long-Run: A Generalization of Preferences in Growth Models, *Journal of Mathematical Economics*, 68C, 80-6.

Bucci, Alberto and Vladimir Matveenko, 2017, Horizontal Differentiation and Economic Growth under non-CES Aggregate Production Function, *Journal* *of Economics*, 120, 1, 1-29.

Cavallari, Lilia, 2013, A Note on Firms' entry, Markup and the Business Cycle, *Economic Modelling*, 35, 528-535.

Colciago, Andrea and Lorenza Rossi, 2015, Firm Dynamics, Endogenous Markups, And The Labor Share Of Income, *Macroeconomic Dynamics*, 19, 6, 1309-31.

Cozzi, Guido, 2017, Combining Semi-endogenous and fully Endogenous Growth: A generalization, *Economics Letters*, 155 (C), 89-91.

Diewert, Walter Erwin, 1971, An Application of the Shephard Duality Theorem: A Generalized Leontief Production Function, *Journal of Political Economy*, 79, 3, 481-507.

Dixon, Huw and Anthony Savagar, 2020, Firm Entry, Excess Capacity and Aggregate Productivity, *European Economic Review*, 121, 103339.

Dunford, Nelson and Jacob Schwartz, 1988, *Linear Operators*, Wiley Classics Library.

Etro, Federico, 2020, Technological Foundations for Dynamic Models with Endogenous Entry, *European Economic Review*, 128, 103517.

Feenstra, Robert, 2018, Restoring the Product Variety and Pro-Competitive Gains from Trade with Heterogeneous Firms and Bounded Productivity, *Journal of International Economics*, 110 (C), 16-27.

Jones, Charles, 1995, R&D-based Models of Economic Growth, *Journal of Political Economy*, 103, 4, 759-84.

Latzer, Helene, Kiminori Matsuyama and Mathieu Parenti, 2020, Reconsidering the Market Size Effect in Innovation and Growth, mimeo, Northwestern University.

Parenti, Mathieu, Philip Ushchev and Jean-Francois Thisse, 2017, Toward a Theory of Monopolistic Competition, *Journal of Economic Theory*, 167 (C), 86-115.

Peretto, Pietro, 1998, Technological Change and Population Growth, *Journal of Economic Growth*, 3, 4, 283-311.

Peretto, Pietro, 2018, Robust Endogenous Growth, *European Economic Review*, 108, 49-77.

Piersanti, Fabio Massimo and Patrizio Tirelli, 2020, Endogenous Productivity Dynamics in a Two-Sector Business Cycle Model, University of Milan Bicocca, WP 434.

Romer, Paul, 1987, Growth based on Increasing Returns due to Specialization, *American Economic Review*, 77, 2, 56-62.

Romer, Paul, 1990, Endogenous Technological Change, *Journal of Political Economy*, 98, 5, S71-102.

Solow, Robert, 1956, A Contribution to the Theory of Economic Growth, *Quarterly Journal of Economics*, 70, 1, 65-94.