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Heterogeneous Fundamentalists and Market Maker Inventories*

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DRAFT: Comments are welcome

Abstract

In this paper we develop an heterogenous agents model of asset price and inventory with a market maker who considers the excess demand of two groups of agents that employ the same trading rule (i.e. fundamentalists) with different beliefs on the fundamental value. The dynamics of our model is driven by a bi-dimensional discrete non-linear map. We show that the market maker has a destabilizing role when she actively manages the inventory. Moreover, inventory share and the distance between agents' beliefs strongly influence the results: market instability and periodic, or even, chaotic price fluctuations can be generated. Finally, we show through simulations that endogenous fluctuations of the fractions of agents may trigger to instability for a larger set of the parameters.

Keywords: heterogeneous agents models; market maker inventory; chaos;
JEL Classification: C61; C63; D84; G12.

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1 Introduction

The market price determination has been historically a crucial issue in the economic literature. Views about the different roles of the so called "specialists" have intensely changed through time since they have been performing increasingly complex functions. On the one hand, the market maker is originally described as 'the broker's broker' and inserted in the "Walrasian auctioneer" framework to describe the price formation process. On the other hand, a "market maker mechanism" has been extensively used to describe the financial markets, when out of the equilibrium exchanges are possible.

Beja and Goldman (1980) are among the first authors that introduced a stylized representation of the market maker[1]. Day and Huang (1990) give also an important contribution on this literature, developing a non-linear behavioral model which achieves chaotic fluctuations around a benchmark fundamental price that may be seen as the bull and bear fluctuations. The market maker reacts to the excess demand by setting the price. She behaves in two different ways: she accumulates (decumulates) stocks in (out) of her inventory in presence of excess supply (demand), like the so-called dealers or liquidity providers, or rather behaves as an active investor maximizing her profits by actively controlling her inventory. These two behaviors may be consistent with each other if a target level of inventory is introduced (Bradfield, 1979).

Madhavan and Smidt (1993) shed new light on the role of specialists' inventory: their basic idea is that when the specialists act as dealers, their quotes induce mean reversion towards a target inventory level, while when they behave as active investors they choose a long-term desired inventory based on portfolio considerations, and may periodically revise this target. Also, they extend the market maker inventory control models incorporating the "asymmetric information effects" combined with level shifts in the target inventory. The existence of a target level redefines the market maker figure as an agent who has a degree of decision on its future market positions.

Since the seminal contribution of Day and Huang (1990), the market maker has been extensively analysed in the framework of the heterogeneous agent models (HAMs). This theoretical approach shows that the behavior of heterogeneous agents may generate complex price dynamics (Kirman, 1991, Lux, 1995, Brock and Hommes, 1997, 1998; Chiarella and He, 2001 or Farmer and Joshi, 2002; Westerhoff and Dieci, 2006) that may replicate stylized facts such as bubbles and crashes, fat tails for the distribution of the returns and volatility clustering.

However, in the HAM's framework, some contributions focus on the impact of the market maker inventory on the price (Gu, 1995; Sethi, 1996; Day, 1997; Franke and Asada, 2008), but none of these seeks to model the market maker as an active investor. To the best of our knowledge, Westerhoff (2003a) is the first that analyses how inventory management of foreign exchange dealers may affect exchange-rate dynamics. Later on, Zhu et al. (2009) develop a model consisting in a market with two different groups of agents (fundamentalists and chartists) plus a market maker acting both as a dealer and an active investor, showing that *'the market maker does not necessarily stabilize the market when actively manages his/her inventory to maximize the profit'* (Zhu et al., 2009 p.3165). Modelling market makers may make the dynamics much easier because a parameter, the market maker reaction coefficient, is added. However we believe it makes models much closer to the real markets (Farmer and Joshi, 2002) and may simplify the analysis (i.e. Hommes et al., 2005).

Most of the HAMs models are characterized by a framework with two assets (a risky and a risk-free asset), different types of traders (i.e. fundamentalists, chartists, noise traders and so on) and sometimes a market maker (for a survey see Hommes, 2006; LeBaron, 2006).

[1]hereafter we use indifferently either specialists or market makers.

By definition, fundamentalists are convinced that prices will return toward their long-run equilibrium values. Hence, if the price is below (above) its fundamental value, they will buy (sell) the asset. Such a trading strategy tends to stabilize the market since prices are pushed toward their equilibrium values. The market impact of fundamental traders is constant over time. Recently, in contrast with the canonical HAM's models, Naimzada and Ricchiuti (2008, 2009, 2012) developed a framework in which the source of instability resides in the interaction of two different groups of agents that use the same trading strategy (all traders are fundamentalists) but have heterogeneous beliefs about the fundamental asset value. We may think about the two different agents' beliefs also within a system that lies outside the financial markets, for example at macro level, which is the case reported for inflation expectations by Mankiw et al. (2003).

The goal of the present paper is twofold. Firstly, starting from the framework developed by Naimzada and Ricchiuti (2009) and in line with the market maker inventory introduced by Zhu et al. (2009), we develop a simple model in which two groups of fundamentalists trade in a financial market with a market maker who actively manages her inventory. Secondly, we study in such a framework the role of heterogeneity and whether the market maker is a stabilizer or not, analysing through simulations both the cases with fixed and endogenously determined fractions.

The remainder of this paper is organized as follows. In section 2 we discuss the asset pricing model with two fundamentalists and the market maker inventory. Section 3 briefly presents the necessary conditions for the existence and local stability of fixed points. In section 4 we use simulations to study the role of heterogeneity also in the case with endogenous fractions of agents. The last section contains the final remarks and suggestions for further investigations.

2 The model

Naimzada and Ricchiuti model (2009) includes a market maker and two archetypal groups of fundamentalists *'who may use one of a number of predictor which they might obtain from financial gurus'* (experts) as in Föllmer et al. (2005). There are two different assets: agents can either invest in a risky asset or in a risk-free asset. The risky asset (e.g stock or stock market index) has a price per share ex-dividend at time t equal to X_t and a (stochastic) dividend process y_t. The risk-free asset is perfectly elastically supplied at the gross return ($R = (1 + r/k) > 1$), where r is equal to the constant risk free rate per annual and k is the frequency of the trading period per year. We define $i = 1, 2$ the *two* groups of agents, and we assume that all the investors choose their own portfolio in a way such that they maximize their expected utility. We denote as z_s the total fixed risky asset supply. The portfolio wealth at $(t + 1)$ is given by:

$$W_{i,t+1} = RW_{i,t} + R_{t+1}q_{i,t} = RW_{i,t} + (X_{t+1} + y_{t+1} - RX_t)\, q_{i,t} \tag{1}$$

where $R_{t+1} = (X_{t+1} + y_{t+1} - RX_t)$ corresponds to the excess return (capital gain/loss) of the risky asset over the trading period $t+1$[2], while $q_{i,t}$ represents the number of shares of the risky asset held in the trading period t by the investor i.

Now, let $E_{i,t}(X_{t+1})$ and $V_{i,t}(X_{t+1})$ be the beliefs or forecasts about the future dividends and the conditional variance of the quantity X_{t+1} respectively. It follows from (1) that:

$$E_{i,t}(W_{t+1}) = RW_{i,t} + E_{i,t}\,(X_{t+1} + y_{t+1} - RX_t)\, q_{i,t}, \tag{2}$$

[2] which is conditionally normally distributed

$$V_{i,t}(W_{t+1}) = q_{i,t}^2 V_{i,t}(R_{t+1}). \tag{3}$$

Let's assume for agents of group i a constant absolute risk aversion utility function equal to $U_i(W) = -e^{-a_i W}$, where a_i represents the - strictly positive and constant - risk aversion coefficient equal for both groups of agents, we assume that $a_i = 1 \, \forall \, i \in [0, \infty]$. By maximizing the expected utility of wealth in trading period $t+1$

$$Max_{q_i,t}\Big[E_{i,t}(W_{i,t+1}) - \frac{a}{2}V_{i,t}(W_{i,t+1})\Big], \tag{4}$$

we obtain the optimal demand function for each group

$$q_{i,t}^* = \frac{E_{i,t}(R_{t+1})}{aV_{i,t}(R_{t+1})} = \frac{E_{i,t}(X_{t+1} + y_{t+1} - RX_t)}{aV_{i,t}(R_{t+1})}. \tag{5}$$

We assume that agents have common expectations on dividends ($E_{i,t}(y_{t+1}) = E_t(y_{t+1}) = \bar{y}$) but different expectations on future prices $E_{i,t}(X_{t+1}) = E_i(X_{t+1}^*) = F_i$ with $i = 1, 2$, where F_i is the fundamental value.

Therefore, to model the excess demand we rewrite the equation (5) adopting the formulation of Day and Huang (1990):

$$q_{i,t} = \delta(F_i^* - P_t), \tag{6}$$

where $P_t = RX_t - \bar{y}$ and $\delta = \frac{1}{a\sigma^2}$ is the positive coefficient of the reaction of investors, a measure of both risk aversion and reaction to mis-pricing of the fundamentalists which we assume, without loss of generality, being equal to 1.

2.1 Inventory

In Naimzada and Ricchiuti (2009), the market maker intervenes clearing the price but she does not manage both her own portfolio and the inventory. In this paper, we consider the two market maker functions - dealer and active investor - as completely segmented. Following Madhavan and Smidt (1993), the market maker - *active investor* - aims to maintain a long-term desired *target inventory position* I^d by demanding in each period the desired inventory plus a share of the previous value of inventory. Let I_t be the specialist's inventory position at time t, then the desired position (I_{t+1}^d) in each period is anything but a share κ of I_t plus the fixed long term target inventory position:

$$I_{t+1}^d = \kappa I_t + I^d, \quad with \quad \kappa \in [0, 1) \tag{7}$$

Eq. (7) represents the market maker demand function.

On the other hand, acting as *dealer*, the market maker provides a required amount of liquidity to the security's market. The market maker inventory at $t+1$ is the desired inventory position in the next trading period plus the total supply of the risky asset minus the investors aggregate optimal demand of the assets at time t:

$$I_{t+1} = I_{t+1}^d + (z_s - z_t^*). \tag{8}$$

Substituting (7) in (8), the equality becomes

$$I_{t+1} = \kappa I_t + I^d + (z_s - z_t^*) \tag{9}$$

with the traders aggregate demand z_t^* at time t being equal to

$$z_t^* = n_{1,t+1}q_1 + (1 - n_{1,t+1})q_2 + \epsilon_t, \quad with \quad n_1 + n_2 = 1, \tag{10}$$

where ϵ_t is the demand error term[3], and $n_{i,t+1}$ is the fraction of agents that follow the expert i. Fractions can be fixed or they can vary according to an adaptive system such as Brock and Hommes (BH, henceforth)(1998). For the analytical results, as in Zhu et al. (2009), we will assume fixed fractions. This assumption will be relaxed in the simulations where we will employ the BH switching mechanism.

Given our assumptions, the market excess demand ED_t for the risky asset in trading period $t+1$ is as follows:

$$ED_t = z_t^* + I_{t+1}^d - z_s \tag{11}$$

where z_t represents the market demand, I_{t+1}^d the *market maker* demand and z_s the supply of the market maker to the outside investors. The other investors adjust their holdings to their optimal demand in trading period $t+1$ by submitting market orders at price P_{t+1}. The market maker adjusts the price so that the return is an increasing function of the market excess demand. If the excess demand ED_t is positive (negative), she increases (decreases) the price:

$$P_{t+1} = P_t + P_t\gamma ED_t = P_t + P_t\gamma[z_t^* + I_{t+1}^d - z_s] \tag{12}$$

where P_t is the asset price at time t and $\gamma > 0$ is the sensitivity of market maker to the excess demand. Finally, the relation that determines the dynamics of the model is obtained by substituting (7) and (10) into (12) and adding the market maker demand:

$$P_{t+1} = P_t + P_t\gamma[n_{1,t+1}q_1 + n_{2,t+1}q_2 + \kappa I_t + I^d - z_s] + \epsilon_t \tag{13}$$

where ϵ_t is a white noise term, i.i.d. normally distributed with mean 0 and variance σ_ϵ^2. The asset price and the inventory dynamics are determined by the following stochastic discrete non-linear dynamical system of equations:

$$\begin{cases} P_{t+1} = P_t[1 + \gamma[n_1(F_1 - P_t) + n_2(F_2 - P_t) + \kappa I_t + I^d - z_s]] + \epsilon_t \\ I_{t+1} = \kappa I_t + I^d + [z_s - [n_1(F_1 - P_t) + n_2(F_2 - P_t)]] + \epsilon_t \end{cases} \tag{14}$$

3 The Deterministic Model: Dynamic Analysis

Let us assume that the system is deterministic. We first calculate the steady states and afterwards we qualitatively work out some properties including the fixed points stability conditions.

3.1 Fixed Points

Proposition 1. *The map (14) has two steady states:*

$$(P_1^*, I_1^*) = \left(0, \frac{z_s + I^d - G}{1 - \kappa}\right) \tag{15}$$

and

$$(P_2^*, I_2^*) = \left(G - z_s + \frac{I^d}{1 - 2\kappa}, \frac{2I^d}{1 - 2\kappa}\right) \tag{16}$$

[3] i.i.d. random variable normally distributed with mean 0 and variance σ_μ^2

with $G = n_1F_1 + n_2F_2$

Proof. The proof is in the Appendix. □

3.2 Local Stability Analysis

The study of the local stability of the equilibria starts with the determination of the Jacobian matrix of the two-dimensional map. The Jacobian matrix of system (14) has the form:

$$J = \begin{bmatrix} 1+\gamma(G-2P_t+\kappa I_t+I^d-z_s) & \gamma\kappa P_t \\ 1 & \kappa \end{bmatrix} \tag{17}$$

Thus, using straightforward algebra the Jacobian matrix of the system (14) at the equilibrium point $E_1(P_1^*, I_1^*)$ is:

$$J(P_1^*, I_1^*) = \begin{bmatrix} 1+\gamma\left(\frac{G+I^d-2\kappa I^d-z_s}{1-\kappa}\right) & 0 \\ 1 & \kappa \end{bmatrix} \tag{18}$$

and from the resulting matrix (18) we work out the following trace and determinant:

$$Tr(J^1) = 1+\gamma\left(\frac{G+I^d-2\kappa I^d-z_s}{1-\kappa}\right)+\kappa \tag{19}$$

$$Det(J^1) = \kappa+\gamma\frac{\kappa}{1-\kappa}\left(G+I^d-2\kappa I^d-z_s\right) \tag{20}$$

Finally, by using Jury's conditions (21) (Jury, 1974) we have conditions for local stability:

$$\begin{cases} 1-TrJ^*+DetJ^*>0 \\ 1+TrJ^*+DetJ^*>0 \\ DetJ^*<1 \end{cases} \tag{21}$$

By substituting trace (19) and determinant (20) in (21) and rearranging the system inequalities we obtain the following stability conditions (22):

$$\begin{cases} \gamma(G+I^d-2G\kappa+(2\kappa-1)z_s)<0 \\ (\kappa^2-1)(2*(\kappa-1)-\gamma(G+I^d-2G\kappa+(2\kappa-1)z_s))>0 \\ (\kappa-1)(1+\kappa(\kappa-2-\gamma(G+I^d-2G\kappa+(2\kappa-1)z_s)))<0 \end{cases} \tag{22}$$

Moreover, the Jacobian evaluated at the second fixed point (P_2^*, I_2^*) is given by:

$$J(P_2^*, I_2^*) = \begin{bmatrix} 1+\gamma\left(z_s-G-\frac{I^d}{1-2\kappa}\right) & \left(G-z_s+\frac{I^d}{1-2\kappa}\right)\kappa\gamma \\ 1 & \kappa \end{bmatrix}, \tag{23}$$

trace and determinant are thus:

$$Tr(J^2) = 1+\gamma\left(z_s-G-\frac{I^d}{1-2\kappa}\right)+\kappa \tag{24}$$

$$Det(J^2) = \kappa+2\kappa\gamma\left(z_s-G-\frac{I^d}{1-2\kappa}\right). \tag{25}$$

The stability conditions for the second fixed point are reported in (26)

$$\begin{cases} \gamma(G + I^d - 2G\kappa + (2\kappa - 1)z_s) > 0 \\ (2\kappa - 1)(2 - 2\kappa - 4\kappa^2 + G\gamma(4\kappa^2 - 1) - \gamma(1 + 2\kappa)(I^d + (2\kappa - 1)z_s)) < 0 \\ \frac{\kappa(2\kappa - 1 + 2\gamma(G + I^d - 2G\kappa + (2\kappa - 1)z_s))}{2\kappa - 1} < 1 \end{cases} \quad (26)$$

We do not have a clear analytical outcome for the stability of the two steady states. Therefore we proceed in the following section through simulations.

4 Numerical analysis

The main purpose of this section is to show the complicated dynamic features of the model through simulations. We calibrate the model according to the characteristics highlighted in our framework and replicate the parameters reported by Zhu et al. (2009). Table 4 shows our initial parameter settings for the simulations. Moreover, we focus our analysis on the steady state with a positive price as shown in eq. 16.

γ	z_s	κ	n_1	n_2	F_1	F_2	I^d
0.18	1	0.1	0.5	0.5	2	3	10

Table 1: Parameter settings for the simulations

We describe how the stability changes as both (i) the sensitivity of the market maker γ and (ii) the distance between the two fundamental values F_1 and F_2 (i.e. the degree of heterogeneity) increase.

4.1 Fixed Fractions

Figures 1 and 2 show respectively the phase plots in price and inventory space and the bifurcation diagram for an increasing γ. The steady state of equation (16) is stable for small values of γ ($\gamma \leq 0.127$), while for increasing values of γ there is a cascade of period-doubling bifurcations that leads to chaos.

To better evaluate the difference between the two fundamentals, it is worth highlighting the evolution of the system from the situation in which there is complete homogeneity ($F_1 = F_2 = F$) with a low γ (0.1) showing the effects of an increasing heterogeneity (an increasing F_2). In Fig. 3 we show the bifurcation diagram of P_t: an increase in the degree of heterogeneity generate instability. When expectations are homogeneous $F_1 = F_2 = 2$ the system is stable. A flip bifurcation arise for $F_2 \approx 12$ and two stable steady states arise. From this points onwards, a cascade of period doubling bifurcations leads the system to chaos. Fig. 4 supports this evidence showing how the inventory fraction held by the market maker plays a key rule for the stability of the system. The parameter basin of attraction shows that there is an inverse relationship between k and the degree of heterogeneity (an increasing F_2): a larger (smaller) distance between the two fundamentals leads to instability for a smaller (larger) κ.

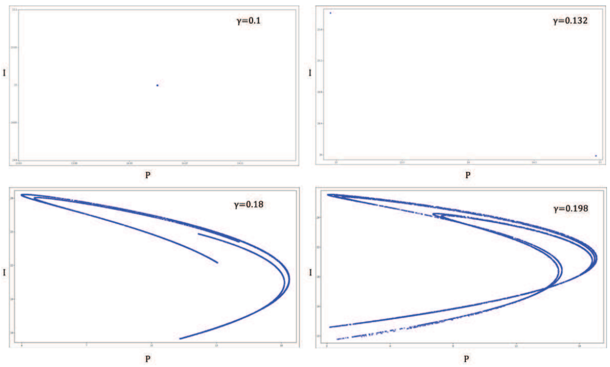


Figure 1: Phase plots of $(P_t,\ I_t)$ for different values of γ

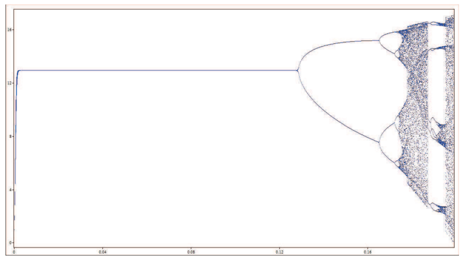


Figure 2: Bifurcation diagram of P_t with $0 < \gamma < 0.196$

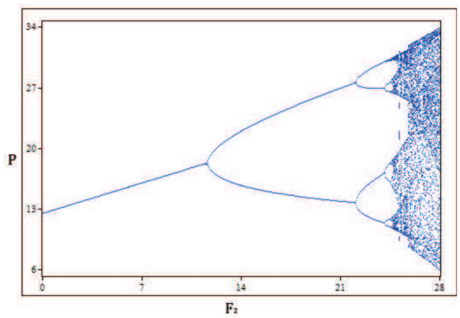


Figure 3: Bifurcation diagram of the degree of heterogeneity variation, for $\gamma = 0.1$

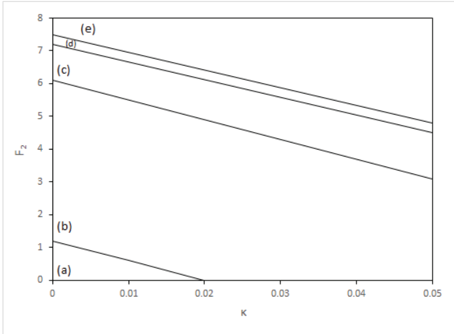


Figure 4: Qualitative basin of attraction of κ and F_2. Local stability region (a), Period-2 cycle (b), further period-doubling cycles (c), (d), (e)

We try to better understand both the activity of the market maker and the incidence of an increasing degree of heterogeneity on the system stability, by analysing time series plots obtained through the simulations (Fig. 5) and summarizing the main descriptive statistics (Tab. 2). In these graphs we add an i.i.d positive stochastic error. In fig 5, we consider the following combinations of parameters $(\gamma, \sigma_\epsilon) = (0.127, 0)$, $(0.127, 0.1)$, $(0.196, 0)$, $(0.196, 0.1)$ holding $F_1 = 2$ and $F_2 = 3$, plotting time series of the price when the system is stable/unstable both in the deterministic (σ_ϵ=0) and stochastic (σ_ϵ=0.1)case. In the top panel (5 (a)) there is a period 2-cycle, adding the noise is added (5 (c)), the two fundamentalist beliefs about the price become more complex, generating larger and irregular fluctuations around the fundamental price. When the activity of the market maker becomes increasingly stronger (5 (b), 5 (d)), the market displays much more complicated dynamics characterized by irregular time series and showing a higher volatility.

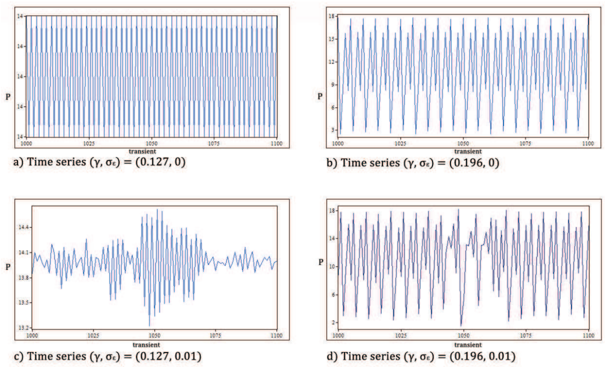
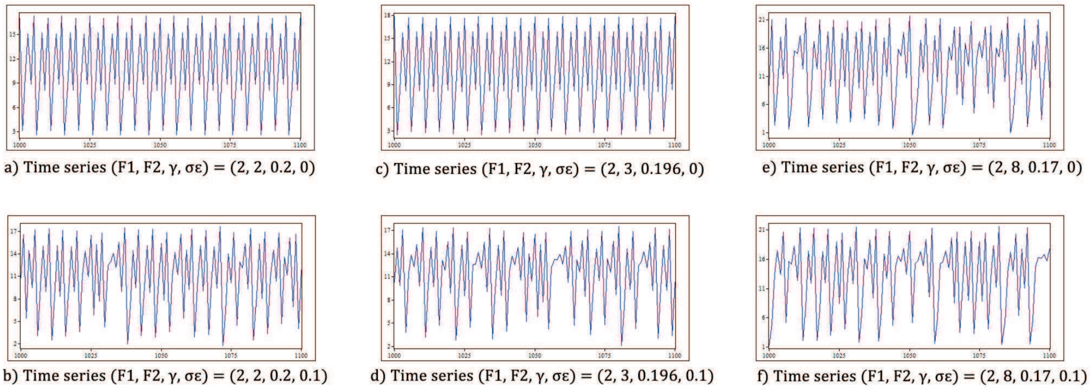


Figure 5: Time series of fundamental price for the specified parameters with noise:(a) and (b); (time series of fundamental price for the specified parameters and without noise: (c) and (d).

Increasingly complex dynamics are shown also in Fig. 6 where 100 consecutive values of the price are plotted for three different sets of values of $(F_1, F_2, \gamma, \sigma_\epsilon)$. In panel (a) F_1, F_2 are equal and time series quasi periodically fluctuate around the mean value of the price. The variability of the time series reflects the increase the degree of heterogeneity, this is observable in panel 6 (c-d) and (e-f) for a much higher degree of heterogeneity. This is the most interesting scenario, because the dynamics shown in Figure 6 are perfectly comparable with those obtained by employing more sophisticated stochastic model. Our simple model is able to generate simulations of some of the most crucial issues happening in the financial markets, in particular the excess volatility. Table 2 summarizes some of the descriptive statistics related to the simulations run over $t = 10,000$ periods. As expected an increasing degree of heterogeneity leads to an increase in the mean and median of the time series as the two fundamental values act as focal points. Compared to the variance of $(F_1; F_2) = (2,2), (2,3)$ the variance of $(F_1; F_2) = (2,8)$ is almost double, this clearly reflects strong excess volatility. Since the kurtosis is always lower than 3 (i.e. the theoretical value of a Normal distribution), the computed time series do not possess fat tails.



a) Time series (F1, F2, γ, σε) = (2, 2, 0.2, 0)
c) Time series (F1, F2, γ, σε) = (2, 3, 0.196, 0)
e) Time series (F1, F2, γ, σε) = (2, 8, 0.17, 0)
b) Time series (F1, F2, γ, σε) = (2, 2, 0.2, 0.1)
d) Time series (F1, F2, γ, σε) = (2, 3, 0.196, 0.1)
f) Time series (F1, F2, γ, σε) = (2, 8, 0.17, 0.1)

Figure 6: Price charts for different degrees of heterogeneity (F_1, F_2), γ values, and in presence (absence) of noise (σ_ϵ=0.1;0)

		$(F_1; F_2)$		
		(2; 2)	(2; 3)	(2; 8)
Mean	$\sigma_\epsilon = 0.1$	10.91	10.97	12.79
	$\sigma_\epsilon = 0$	10.58	10.73	12.69
Median	$\sigma_\epsilon = 0.1$	10.38	10.11	13.49
	$\sigma_\epsilon = 0$	10.58	10.73	12.69
Variance	$\sigma_\epsilon = 0.1$	24.08	28.27	41.1
	$\sigma_\epsilon = 0$	26.07	29.56	41.59
Kurtosis	$\sigma_\epsilon = 0.1$	1.85	1.74	1.82
	$\sigma_\epsilon = 0$	1.73	1.63	1.78
Skewness	$\sigma_\epsilon = 0.1$	-0.29	-0.19	-0.31
	$\sigma_\epsilon = 0$	-0.13	-0.06	-0.26

Table 2: Descriptive statistics of the simulated time series

As suggested by Westerhoff and Franke (2009), in order to determine the ability of the model to replicate empirical long memory effects, we perform the autocorrelation function plots for all the combination of prices, inventory and fundamental values introduced before. Figure (9) and (10) in Appendix B depict the autocorrelation functions of prices and inventories. In eight out of ten plots it is revealed the presence of significant correlation. However, for higher degree of heterogeneity ($F_1 = 2$, $F_2 = 8$) the model successfully reproduces the stylized facts of uncorrelated prices and inventory.

4.2 Endogenous Fractions with BH

In this section we analyse "through" simulation a generalized version of the above model when fractions of agents are endogenous. We assume that agents can switch from guru to the other following an adaptive belief system la Brock and Hommes (1997, 1998). Specifically, the fractions of agents n_i is:

$$n_{i,t+1} = \frac{exp[-\beta(F_1 - P_t)^2]}{exp[-\beta(F_1 - P_t)^2] + exp[-\beta(F_2 - P_t)^2]}. \tag{27}$$

where β is the so-called intensity of choice, a parameter which assesses how quickly agents *switch* between the two predictions. Substituting (27) in (13) we obtain the following general map:

$$\begin{cases} P_{t+1} = P_t[1 + \gamma[n_{1,t+1}(F_1 - P_t) + n_{2,t+1}(F_2 - P_t) + \kappa I_t + I^d - z_s]] + \mu_t \\ I_{t+1} = \kappa I_t + I^d + [z_s - [n_{1,t+1}(F_1 - P_t) + n_{2,t+1}(F_2 - P_t)]] + \mu_t \end{cases} \tag{28}$$

In Fig. 7 we replicate the parameter basin of attraction of the fig. 4 with a small $\beta = 0.1$. It is worth noting that differently from Fig. 4, for a very low κ stability occurs also when fractions are homogeneous. A further increase in the parameters value leads to instability. As we expect, if β increases, the set of parameters for which there is instability is less restrictive 8. The table 3 could be comparable with Tab. 2. However, from the statistical point of view the conclusions are the same: the greater the heterogeneity among the agents the greater the mean / median / variance of the series.

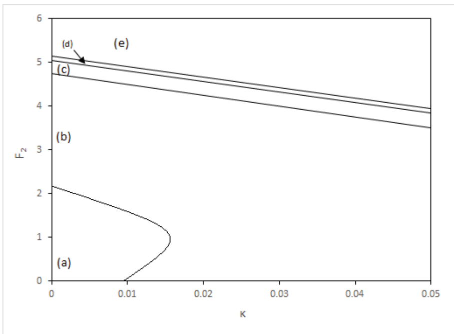


Figure 7: Qualitative basin of attraction for κ and F_2 with BH. Local stability region (a), Period-2 cycle (b), further period-doubling cycles (c), (d), (e)

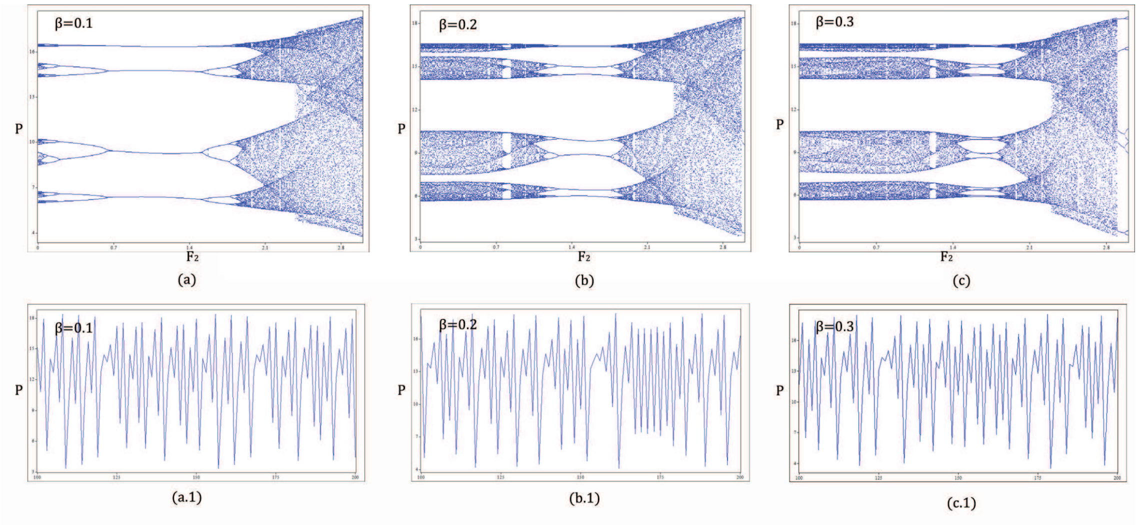


Figure 8: P_t Bifurcations plots of degrees of heterogeneity (ΔF) variation for $\beta = (0.1, 0.2, 0.3)$, with n_i endogenous. Time series charts are computed for $(F_1, F_2) = (2, 3)$

Descriptive Statistic	Noise	$(F_1; F_2)$								
		$\beta = 0.1$			$\beta = 0.2$			$\beta = 0.3$		
		(2;2)	(2;3)	(2;8)	(2;2)	(2;3)	(2;8)	(2;2)	(2;3)	(2;8)
Mean	$\sigma = 0.1$	11.64	11.85	15.7	10.32	11.51	15.65	10.18	11.26	15.48
	$\sigma = 0$	11.65	11.39	15.06	10.21	11.13	14.35	10.39	11.32	15.13
Median	$\sigma = 0.1$	12.36	11.72	16.47	8.04	11.02	16.43	8.75	11.7	16.24
	$\sigma = 0$	12.39	10.06	16.29	10.12	9.38	12.72	10.86	11.63	16.79
Variance	$\sigma = 0.1$	18.15	25.45	49.02	27.59	28.83	49.56	28.59	31.14	51.7
	$\sigma = 0$	18.04	28.77	50.03	29.06	31.29	60.18	27.8	30.61	55.86
Skewness	$\sigma = 0.1$	-0.53	-0.33	-0.24	-0.05	-0.23	-0.23	-0.04	-0.28	-0.24
	$\sigma = 0$	-0.53	-0.15	-0.21	-0.24	-0.07	0.06	-0.27	-0.27	-0.32
Kurtosis	$\sigma = 0.1$	2.16	1.9	1.69	1.62	1.77	1.65	1.62	1.8	1.66
	$\sigma = 0$	2.15	1.75	1.65	1.77	1.65	1.55	1.79	1.79	1.77

Table 3: Descriptive statistics of the simulated time series with n_i *a la Brock and Hommes* for different values of β and different agents beliefs

5 Conclusions

This paper contributes to the development of financial market modelling and asset price dynamics with heterogeneous agents. We develop a model of asset price and inventory in a scenario where a market maker sets the price to clear the market. In doing so, the market maker considers the excess demand of two groups of agents that employ the same trading rule (i.e. two fundamentalists) but with different beliefs on the fundamental prices. Moreover, the market maker has a double role: she provides liquidity and acts as an active investor. When κ is null, the model is equal to Naimzada and Ricchiuti (2014). In contrast with the canonical literature and similarly to Naimzada and Ricchiuti (2008, 2009, 2014) we show that interactions between agents with homogeous trading rules can lead to market instability. The active role of the market maker can be one of the causes of instability in the financial markets: a higher inventory share leads to instability. Finally, buffeted with dynamic noise, this model may also replicate some important stylized facts of financial markets such as excess volatility and volatility clustering.

Further improvements to our behavioural financial model may consist in analysing a case when more actors with a long memory are introduced in the model. This can be pursued by introducing in the model a trend follower. In such a case it will be possible to analyse the interactions between many views within the economic system and their survival in an evolutionary environment based on historical data. In order to control for market distortions and price volatility at the same time it would be interesting to introduce price limiters as attempted by Westerhoff (2003b). Moreover, the model can be improved by comparing this model with other where the market mechanism is different (Walrasian auctioneer for example) as in Anufriev and Panchenko (2009) and observing how the simulated time series behaviour varies across the different specifications.

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6 Appendix A

In this appendix, we explain for easy reference the mathematical procedures used in our analysis, in particular we develop a step by step procedure for the fixed points computation.

Step 1) The system of prices and inventory is:

$$\begin{cases} P_{t+1} = P_t[1 + \gamma[n_1(F_1 - P_t) + n_2(F_2 - P_t) + \kappa I_t + I^d - z_s]] + \mu_t \\ I_{t+1} = \kappa I_t + I^d + [z_s - [n_1(F_1 - P_t) + n_2(F_2 - P_t)]] + \mu_t \end{cases} \tag{29}$$

Step 2) Setting P^* and I^* as follows:

$P_t = P_{t+1} = P^*$
$I_t = I_{t+1} = I^*$

with $G = n_1F_1 + n_2F_2$

we obtain the following system of equations:

$$\begin{cases} P^* = P^* + P^*\gamma[n_1(F_1 - P_t) + n_2(F_2 - P_t) + \kappa I_t + I^d - z_s]] + \mu_t \\ I^* = \frac{z_s - (n_1F_1 - n_1P_t + n_2F_2 - n_2P_t) + I^d}{1-\kappa} \end{cases} \tag{30}$$

Step 3) **fixed points** are obtained by setting $P^* = 0$ and $G = n_1F_1 + n_2F_2$

$$\begin{cases} P_1^* = 0 \\ I_1^* = \frac{z_s + I^d - G}{1-\kappa} \end{cases} \tag{31}$$

Step 4) the second solution of the system is computed by solving the system for P^* and I^*.

$$\begin{cases} G - P^* + \kappa I^* + I^d - z_s = 0 \\ I^* = \frac{1}{1-\kappa}\left[I^d + z_s - G + P^*\right] \end{cases} \tag{32}$$

Step 5) We substitute I^* in the first equation:

$$\begin{cases} P^* = G + \frac{\kappa}{1-\kappa}\left[I^d + z_s - G + P^*\right] + I^d - z_s \\ I^* = \frac{1}{1-\kappa}\left[I^d + z_s - G + P^*\right] \end{cases} \tag{33}$$

Step 6) With further rearrangements we obtain:

$$\begin{cases} \frac{(1-2\kappa)G - (1-2\kappa)z_s + I^d}{1-\kappa} = \left(\frac{1-2\kappa}{2-\kappa}\right) P^* \\ I^*\left(1 - \frac{\kappa}{1-\kappa}\right) = \left(\frac{1}{1-\kappa}\right) 2I^d \end{cases} \tag{34}$$

Step 7) which leads to the second set of fixed points:

$$\begin{cases} P_2^* = G - z_s + \frac{I^d}{1-2\kappa} \\ I_2^* = \frac{2I^d}{1-2\kappa} \end{cases} \tag{35}$$

7 Appendix B

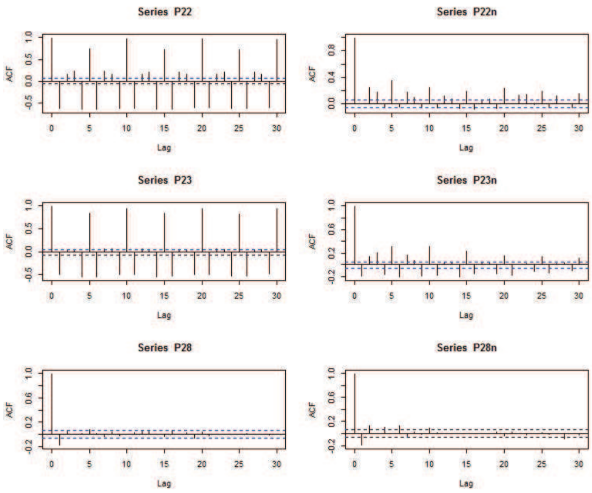

Figure 9: ACF of P_t values, for different (F_1, F_2) combinations and in presence (absence) of noise

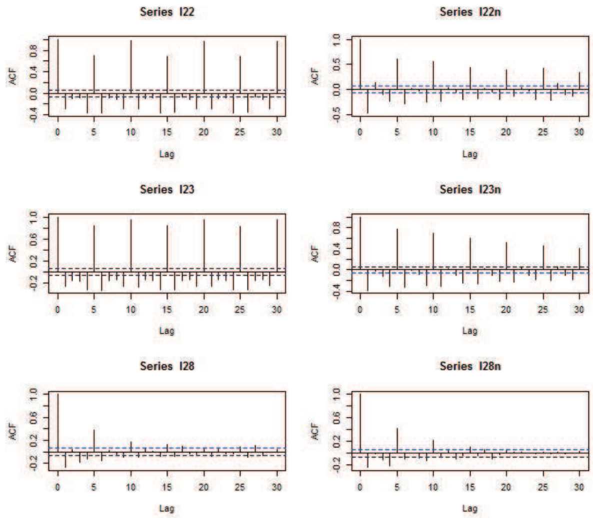

Figure 10: ACF of I_t values, for different (F_1, F_2) combinations and in presence (absence) of noise