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Information revelation in procurement auctions with two-sided asymmetric information*

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Abstract

A buyer needs to procure a good from either of two potential suppliers offering differentiated products and with privately observed costs. The buyer privately observes the own valuations for the products and (ex ante) decides how much of this information should be revealed to suppliers before they play a first score auction. We show that the more significant is each supplier's private information on the own cost, the less information the buyer should reveal. Part of our analysis is linked to the comparison between a first and a second price auction in an asymmetric setup with a distribution shift.

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1 Introduction

In this paper we study a procurement auction in which the auctioneer is a (male) buyer interested in purchasing an object that can be supplied by two different (female) suppliers; each supplier is privately informed about her own production cost. The suppliers offer differentiated products, and the buyer is interested both in the price he pays and in the degree of fitness of a product with his own needs – the latter is a sort of "quality assessment" of the object from the buyer's subjective point of view.[1] We assume that the buyer's assessments of the products' qualities are not observed by suppliers, and we inquire how much (if any) of his assessments the buyer should reveal to suppliers before the bidding process. Our main result is that (under suitable assumptions on the distributions of costs and qualities) the more significant is each supplier's private information on the own cost, the less information on qualities the buyer should reveal.

The literature on procurement when quality matters often analyzes multidimensional auction models in which the buyer announces a scoring rule, and then each bidder makes a multi-dimensional bid specifying both a price and quality level(s): see Che (1993), Branco (1997), Asker and Cantillon (2008, 2010). However, a few papers consider settings in which the buyer subjectively evaluates the quality of each product, and inquire the impact of different information disclosure strategies on the outcome of the auction: see for instance Gal-Or et al. (2007), Rezende (2009), Kostamis et al. (2009), and Kaplan (2011) for theoretical analyses; Haruvy and Katok (2010) and Thomas and Wilson (2011) rely on experimental methods for some specific parameter values.

These issues are relevant, for instance, in electronic procurement auctions in which buyers often evaluate the qualities of different offers according to their own individual tastes, and the influence of non-price attributes is proved by the common use of "non-binding" auctions, in which a buyer is not bound to select a bidder who submitted the lowest price (as documented by the above mentioned papers). A further application of these family of models is represented by conservation auctions. These auctions are competitive mechanisms adopted by some governments (see for instance the Conservation Reserve Program in the US and the Bush Tender Program in Australia) to allocate financial subsidies to farmers in exchange for the implementation of natural resource management programs on their lands. In that context, each farmer privately knows his opportunity cost from joining the program, while the regulator privately knows the program's environmental benefits on each specific land (see Chan et al., 2003). Cason et al. (2003) study experimentally how different disclosure policies of the regulator's information affect the outcome.

The starting point for our analysis is a model studied by Gal-Or et al. (2007) (GGD henceforth), in which the products' qualities are realization of i.i.d. random variables that the buyer privately observes. A first score auction is held in which each supplier bids a price, and the product with the highest difference between quality and price is selected. However, before observing the qualities the buyer commits to one of three possible information revelation policies: public revelation (PU henceforth), private revelation (PR), and concealment (C). Under policy PU, before the auction the buyer publicly discloses his quality assessments for each product; under policy PR, he reveals to each supplier only the quality of the supplier's product; under policy C, he does not reveal any information. Hence, the stage of information revelation is an additional (intermediate) stage with respect to a standard auction procedure.[2] Clearly, the information

[1] In some cases the buyer's assessments take into account some features of a supplier i – such as the supplier's reliability and logistic costs – as long as they are going to affect the buyer's payoff from choosing the product of supplier i.

[2] Notice that (as already remarked by GGD) PR does not require private communication from the buyer to each supplier. Precisely, in order to implement policy PR it suffices that the buyer announces the different attributes of a product he takes into account and the weights he assigns to each attribute in order to determine the degree of fit. After receiving this information, each supplier can determine the quality of the own product as seen by the buyer.

a supplier receives before the auction affects her bidding, and thus the buyer chooses a policy in view of the ensuing suppliers' behavior in the auction. Assuming that suppliers are risk neutral and have identical and commonly known costs, GGD prove that the buyer is indifferent between PR and PU, and prefers these policies to C under suitable restrictions on the distribution of qualities.

We assume that there are only two suppliers, and introduce in this environment a privately observed production cost for each supplier, which can be high (c_H) or low (c_L) with $\sigma \equiv c_H - c_L$.[3] In our model therefore each agent holds some relevant private information, which considerably complicates the analysis with respect to the setting of GGD, especially for policy C. As a consequence, in our study of this policy we need to assume that each product's quality is uniformly distributed over an interval, with $\theta > 0$ denoting the interval length.[4]

We derive equilibrium bidding under each of the three information policies, and then study the buyer's preferences over the policies. Our main result states that the buyer's preferred policy is PU for small values of the ratio σ/θ, is PR for intermediate values of σ/θ, and is C for large values of σ/θ. Therefore, the amount of information the buyer should reveal (weakly) decreases monotonically with respect to σ/θ: the more relevant is the suppliers' private information on costs, with respect to the maximal quality difference between products, the less information on qualities the buyer should reveal to suppliers.

In order to discuss this result for the comparison between PU and PR, we notice that in both these policies each supplier learns the quality of the own product, and this makes the first score procurement auction equivalent to a first price auction (FPA henceforth) with two bidders in which the auctioneer is the seller of an object, and for each bidder i the value of the object is given by the difference between her quality q_i and her cost c_i. Indeed, when supplier i with quality q_i chooses a bid p_i, she implicitly offers a score $q_i - p_i$ to the buyer. Thus we can express her choice problem in terms of selecting a score, and in case of victory her payoff is given by her valuation, $q_i - c_i$, minus the score offered, like in a standard FPA.

Under policy PR the suppliers' valuations are i.i.d., thus standard auction results apply. In particular, the supplier with the highest value wins, and (by revenue equivalence) the buyer's payoff is equal to the expected second highest valuation, as in a second price auction (SPA henceforth). Under policy PU, given q_1, q_2 announced by the buyer – with $q_2 > q_1$ to fix the ideas – the set of possible valuations is $\{q_1 - c_H, q_1 - c_L\}$ for bidder 1 and $\{q_2 - c_H, q_2 - c_L\}$ for bidder 2. This is an asymmetric auction environment in which the asymmetry is generated by a distribution shift with size $q_2 - q_1$. For the purposes of comparison we can view the buyer's payoff under PR, given q_1, q_2, as the expected second highest valuation in the above asymmetric setting. Therefore the ranking between PR and PU is closely related to the ranking between the FPA and the SPA in an asymmetric setting with a distribution shift. For this sort of asymmetry, Maskin and Riley (2000) and Kirkegaard (2011) prove that with continuously distributed valuations (and under some regularity conditions) the FPA is superior to the SPA. However, the valuations in our setting have binary supports and we obtain an opposite result for the case of a small shift. In fact, somewhat counterintuitively, a shift which is small with respect to σ reduces the buyer's payoff in PU (whereas it increases the buyer's payoff in PR) because it induces supplier 1 (the disadvantaged supplier) to bid less aggressively.[5] This suggests that PR is better than PU for a large σ/θ. On the other hand, under PU a

[3] Colucci et al. (2011) consider the framework of GGD but assume that suppliers have different production costs, which are still common knowledge. Thus suppliers are asymmetric ex ante.

[4] With reference to policy C in a setting similar to our, Thomas and Wilson (2011) claim that "theoretical characterization of equilibrium behavior in the settings we consider remains an open problem" to explain their use of experimental methods.

[5] This fact is explored in detail in Doni and Menicucci (2011), which analyze equilibrium bidding in the FPA with two bidders when the valuations are asymmetrically distributed with binary supports. For the same setting, they compare the seller's revenue in the FPA with the revenue in the SPA.

large shift with respect to σ gives supplier 2 (the preferred supplier), a large advantage with respect to supplier 1, and this induces her to bid aggressively enough to outbid supplier 1 whatever are the realized costs. This is the so-called "Getty effect", described in Maskin and Riley (2000), and it makes the payoff of the buyer under PU larger than under PR for a small σ/θ (from GGD we know that PR is better than C for small σ/θ, thus PU is superior to C as well).

When σ/θ is large, C emerges as the optimal policy because it is effective in controlling suppliers' rents. Precisely, for a large σ/θ under policy C a supplier with cost c_L always wins when facing a supplier with cost c_H – as a consequence the highest valuation supplier always wins – and in Subsection 3.1 we show that this makes the suppliers' rents independent of σ/θ.[6] Conversely, under PR standard auction theory suggests that suppliers' rents are given by the expected difference between the highest and the second highest valuation, and considering the states of the world in which suppliers have different costs reveals that this expectation is increasing in σ/θ. Since C and PR generate the same social surplus as the winner is always efficiently selected, it is intuitive that C is superior to PR (and superior to PU as well, as PR is better than PU for a large σ/θ).

The rest of the paper is organized as follows. Next section describes the model, Section 3 derives the suppliers' equilibrium behavior for each information policy, and Section 4 examines the buyer's preferences over the three polices. Section 5 concludes offering some suggestions for further research. The appendix contains the proofs of all our results.

2 The model

In our setting a male buyer denoted with B needs to buy a certain object (for instance, an industrial firm needs to procure an input) and faces two female suppliers which can provide the object. Supplier i, for $i = 1, 2$, privately observes the own production cost $c_i \in \{c_L, c_H\}$; we use $\sigma \equiv c_H - c_L > 0$ to denote the difference between c_H and c_L. Furthermore, (c_1, c_2) are i.i.d. with $\lambda \equiv \Pr\{c_i = c_L\} \in (0, 1)$.

The products offered by the suppliers are differentiated and for each supplier i there is a parameter $q_i \in [\underline{q}, \bar{q}]$, with $\theta \equiv \bar{q} - \underline{q} > 0$, which represents the degree of fitness of i's product with B's needs; in a sense, q_i represents the quality of product i from the subjective point of view of B. Precisely, if B buys the object offered by supplier i and pays p_i, then his payoff is $q_i - p_i$; we use s_i to denote the difference between q_i and p_i, which we call the *score* offered by supplier i to B. The buyer is risk neutral and uses a *first score auction* in which suppliers simultaneously submit bids $p_1 \geq 0, p_2 \geq 0$, and then B buys product i such that $q_i - p_i > q_j - p_j$ (that is such that $s_i > s_j$),[7] paying p_i to supplier i and nothing to j. We suppose that each supplier i is risk neutral and that she wants to maximize $(p_i - c_i)$ times her probability of winning. For each supplier i we define $v_i \equiv q_i - c_i$ as the *value* of supplier i; thus v_i is the social surplus which is generated if B buys product i.

Before running the auction, B observes the values q_1, q_2 but suppliers do not: each supplier views q_1, q_2 as uniformly distributed over $Q \equiv [\underline{q}, \bar{q}] \times [\underline{q}, \bar{q}]$,[8] and stochastically independent of (c_1, c_2). Before observing q_1, q_2, B has the same beliefs as the suppliers about q_1, q_2, and we inquire whether at this stage B should commit to a policy of no information revelation (concealment policy, denoted by C), or to a policy in which he will reveal to each supplier i only the value of q_i (private revelation policy, denoted by PR), or to a

[6] In fact, this property implies that no pure-strategy equilibrium exists under policy C if σ/θ is sufficiently large.

[7] In order to fix the ideas, we suppose that each supplier wins with probability $\frac{1}{2}$ in case of tie. However, the equilibrium outcomes we find do not depend on this assumption.

[8] Some of our results (Propositions 2, 3 and 5) do not depend on this assumption.

policy in which he will publicly reveal both q_1, q_2 (public revelation policy, denoted by PU). Our main objective is to find out the information revelation policy which is most profitable for B.[9]

In order to summarize, we consider a game with the following timing:

- Stage one: B chooses an information revelation policy.
- Stage two: Nature selects qualities (q_1, q_2) and costs (c_1, c_2); B observes (q_1, q_2), supplier i observes c_i, for $i = 1, 2$.
- Stage three: B sends a message to each supplier, consistently with his choice at stage one.
- Stage four: Suppliers bid in the auction and B selects the winner.

In some cases our results are conveniently described by using the ratio σ/θ (as in the introduction) and thus we define $\omega \equiv \frac{\sigma}{\theta}$.

3 Equilibrium under different information policies

3.1 Concealment

Under the policy of concealment, no supplier receives any information about q_1, q_2. However, each supplier (privately) observes her own cost before bidding, and therefore her bid is a function of the cost. Precisely, we use p_{iL}, p_{iH} to denote the bid of supplier i if she has cost c_L (i.e., if she is type L) and her bid if she has cost c_H (i.e., if she is type H), respectively.

As we mentioned in Section 2, given qualities (q_i, q_j) and bids (p_i, p_j), supplier i wins if $q_i - q_j > p_i - p_j$. Therefore, in order to obtain an expression for the payoff function of supplier i, it is useful to define the random variable $t \equiv q_j - q_i$,[10] with support $[-\theta, \theta]$ and the following c.d.f. F and density $f = F'$:

$$F(t) = \begin{cases} \frac{1}{2} + \frac{1}{\theta}t + \frac{1}{2\theta^2}t^2 & \text{for } t \in [-\theta, 0] \\ \frac{1}{2} + \frac{1}{\theta}t - \frac{1}{2\theta^2}t^2 & \text{for } t \in (0, \theta] \end{cases} \qquad f(t) = \begin{cases} \frac{1}{\theta} + \frac{1}{\theta^2}t & \text{for } t \in [-\theta, 0] \\ \frac{1}{\theta} - \frac{1}{\theta^2}t & \text{for } t \in (0, \theta] \end{cases} \tag{1}$$

Now consider type k of supplier i, for $k = L, H$, and notice that bidding p_{ik} gives her a probability of winning against type L of supplier $j \neq i$ equal to $\Pr\{t < p_{jL} - p_{ik}\} = F(p_{jL} - p_{ik})$; the probability of winning against type H of supplier j is $\Pr\{t < p_{jH} - p_{ik}\} = F(p_{jH} - p_{ik})$. As a consequence, the payoff function of type k of supplier i is

$$(p_{ik} - c_k)[\lambda F(p_{jL} - p_{ik}) + (1 - \lambda)F(p_{jH} - p_{ik})] \qquad \text{for} \quad j \neq i, \quad k = L, H \tag{2}$$

Since suppliers are ex ante symmetric, we focus on symmetric Bayes-Nash equilibria (BNE in the following), in which both type L of supplier 1 and type L of supplier 2 submit the same bid p_L and both type H of supplier 1 and type H of supplier 2 submit the same bid p_H. We find a (unique) pure-strategy BNE if the difference σ between c_H and c_L is not too large, that is if $\sigma < \frac{1}{\lambda}\theta$; in such a case $p_H - p_L < \theta$, which means that a supplier with type H wins with positive probability when her opponent has type L. If instead $\sigma \geq \frac{1}{\lambda}\theta$, then we find a pure-strategy BNE if (λ, σ) belongs to the set

$$\mathcal{C} \equiv \left\{(\lambda, \sigma) : \lambda \in (\frac{2}{5}, 1] \quad \text{and} \quad \max\{\frac{1}{\lambda}, \frac{9}{5}\lambda - \frac{3}{10}\}\theta \leq \sigma \leq \frac{5\lambda + 1}{3\lambda}\theta\right\} \tag{3}$$

[9] We suppose that $\underline{q}$ is sufficiently large to make B's payoff positive under B's most convenient policy.

[10] Since q_i, q_j are indentically distributed, the distribution of $q_j - q_i$ is identical to the distribution of $q_i - q_j$.

Moreover, for the special case in which $\lambda=\frac{1}{2}$ and $\sigma\in\{\frac{5}{2}\theta,3\theta,\frac{7}{2}\theta\}$ we described a mixed BNE. We discuss this result after the proposition, in which we denote with U_C^B the buyer's expected payoff in equilibrium.

Proposition 1 *Consider policy C.*
(i) For the case in which $\sigma<\frac{1}{\lambda}\theta$, there exists a unique symmetric BNE and is described as follows:

- *If $\lambda=\frac{1}{2}$, then*

$$\begin{aligned} p_L &= c_L+\frac{1}{4}\sigma+\frac{2\theta^2}{4\theta-\sigma}, \qquad p_H=c_H+\frac{4\theta^2+(2\theta-\sigma)^2}{4(4\theta-\sigma)}=p_L+\frac{1}{2}\sigma \\ U_C^B &= \underline{q}-c_L+\frac{64\theta^4-256\theta^3\sigma+72\theta^2\sigma^2-10\sigma^3\theta+\sigma^4}{96\theta^2\,(4\theta-\sigma)} \end{aligned} \tag{4}$$

- *If $\lambda\neq\frac{1}{2}$, then*

$$\begin{aligned} p_L &= c_L+\frac{(1+2\lambda)\Delta^2-2\,(2\theta+\lambda\sigma)\,\Delta+2\theta\sigma}{2\,(1-2\lambda)\,\Delta}, \qquad p_H=p_L+\Delta \\ U_C^B &= \underline{q}-p_L+\frac{2}{3}\theta-(1-\lambda)\,\Delta\frac{3\theta^2-3\lambda\Delta\theta+\lambda\Delta^2}{3\theta^2} \end{aligned} \tag{5}$$

in which Δ is the unique solution in $(0,\theta)$ to the equation

$$4\lambda\,(1-\lambda)\,\Delta^3-[3\theta+4\lambda\,(1-\lambda)\,\theta+2\lambda\,(1-\lambda)\,\sigma]\Delta^2+(5\theta-2\lambda\theta+2\sigma)\,\theta\Delta-2\theta^2\sigma=0 \tag{6}$$

(ii) For the case in which $(\lambda,\sigma)\in\mathcal{C}$ there exists a symmetric BNE such that $p_H-p_L\geq\theta$ and is described as follows:

$$\begin{aligned} p_L &= c_L+(\frac{1}{\lambda}-\frac{1}{2})\theta, \qquad p_H=c_H+\frac{1}{2}\theta \\ U_C^B &= \underline{q}-c_L-\frac{1}{6}(4\lambda^2-16\lambda+11)\theta-(1-\lambda)^2\sigma \end{aligned} \tag{7}$$

(iii) If $\lambda=\frac{1}{2}$ and $\sigma\in\{\frac{5}{2}\theta,3\theta,\frac{7}{2}\theta\}$, then there exists a symmetric mixed-strategy BNE which is characterized by three bids p_L^a,p_L^b,p_H and a probability $\mu\in(0,1)$ such that each type L bids p_L^a with probability μ, bids p_L^b with probability $1-\mu$, and each type H bids p_H. The equilibrium values of p_L^a,p_L^b,μ,p_H are

ω	p_L^a	p_L^b	μ	p_H	U_C^B
$\frac{5}{2}$	$c_L+1.515598\theta$	$c_L+2.087448\theta$	0.98747	$c_H+0.49995\theta$	$\underline{q}-c_L-1.308088\theta$
3	$c_L+1.792730\theta$	$c_L+2.587446\theta$	0.84623	$c_H+0.493555\theta$	$\underline{q}-c_L-1.711140\theta$
$\frac{7}{2}$	$c_L+2.121458\theta$	$c_L+3.070966\theta$	0.75271	$c_H+0.491099\theta$	$\underline{q}-c_L-2.152948\theta$

The setting studied by GGD is such that $c_H=c_L$, that is $\sigma=0$, and they obtain $p_L=p_H=c_L+\frac{1}{2}\theta$. In order to compare the result of GGD with Proposition 1 it is useful to think of c_L as fixed while $c_H=c_L+\sigma>c_L$. Furthermore, in this discussion we focus on the case of $\lambda=\frac{1}{2}$ since then equilibrium bids have simple expressions.

From (4) it is straightforward to see that increasing σ increases the bid of both type L and type H; in fact, it is intuitive that p_H increases as σ increases, and moreover also p_L increases since bids are strategic complements. Furthermore, the suppliers' ex ante expected payoff increases with σ. From the viewpoint of B, higher bids obviously reduce his payoff and indeed U_C^B is decreasing in σ. Regarding social surplus, we notice that $p_H-p_L=\frac{1}{2}\sigma$, which means that the difference between the bid of type H and the bid of type L is smaller than the cost difference.[11] This generates an efficiency loss from a social point of view since it is not always the case that the supplier with the highest value wins. Precisely, when $c_1 = c_L$ and $c_2 = c_H$ (a symmetric argument applies when $c_1 = c_H$, $c_2 = c_L$) supplier 2 wins as long as q_1, q_2 satisfy $q_2 - p_H > q_1 - p_L$, which reduces to $q_2 > q_1 + \frac{1}{2}\sigma$, although 2 has the highest value if and only if $q_2 > q_1 + \sigma$, and the former condition is satisfied more often than the latter. Thus supplier 2 wins too often from a social point of view.

On the other hand, it is intuitive that a sufficiently large σ induces a large difference in bids such that a type L always wins against a type H, that is $p_H - p_L \geq \theta$. In fact, this occurs if (λ, σ) belongs to the set $\mathcal{C}$ defined in (3) (in this case the equilibrium prices – in (7) – are simple enough that we do not need to restrict to $\lambda = \frac{1}{2}$), which requires that σ is larger than $\frac{\theta}{\lambda}$, but not too larger. When $(\lambda, \sigma) \in \mathcal{C}$ and $c_1 = c_L$, $c_2 = c_H$, for any $(q_1, q_2) \in Q$ supplier 1 has a higher value than supplier 2 and the inequality $q_1 - p_L > q_2 - p_H$ holds for any q_1, q_2, which means that a supplier of type L certainly wins against a supplier with type H; thus social surplus is maximized. However, we can prove the existence of a BNE with this feature only if $(\lambda, \sigma) \in \mathcal{C}$. In order to see why, consider type L and notice that (i) bidding p_L gives her a probability of winning equal to $\frac{\lambda}{2} + 1 - \lambda$ (she wins with probability $\frac{1}{2}$ against type L, and with probability 1 against type H); (ii) on the other hand, bidding $p_H - \theta (> p_L)$ yields a probability of winning at least equal to $1 - \lambda$ (she still wins with probability 1 against type H); (iii) if σ is large and λ is small, then $p_H - \theta$ is significantly larger than p_L and the probability of winning has decreased only slightly; this makes bidding $p_H - \theta$ a profitable deviation for type L.[12]

In addition, when λ is large we need that σ is sufficiently larger than $\frac{\theta}{\lambda}$ because otherwise there exists a profitable deviation for type H which consists of bidding slightly above c_H. Precisely, for a large λ the equilibrium payoff for a type H is small since he wins with positive probability only if his opponent has type H (an event with probability $1 - \lambda$). If instead he bids below $p_L + \theta$, then he wins with positive probability also if his opponent has type L, and moreover increases his probability of winning against type H. Such a behavior is profitable for type H if c_H is sufficiently lower than $p_L + \theta$; the inequality $\sigma > (\frac{9}{5} - \frac{3}{10}\lambda)\theta$ guarantees that c_H is large enough to make unprofitable the above described deviation. Finally, we notice that the suppliers' payoffs do not depend on σ if $(\lambda, \sigma) \in \mathcal{C}$, thus an increase in σ in $\mathcal{C}$ has the only effect of reducing B's payoff.

When (λ, σ), an important feature of the BNE is that the suppliers' rents are constant with respect to σ: the payoff of type L is $(\frac{1}{4}\lambda + \frac{1}{\lambda} - 1)\theta$ and the payoff of type H is $\frac{1-\lambda}{4}\theta$. This occurs because $p_H - p_L > \theta$ implies that (i) supplier 1_L wins against 2_H for any p_{1L} close to p_L, and thus p_L does not depend on c_H nor on σ since small changes in the bid, such as those considered by the first order condition, do not affect the probability for 1_L to win against 2_H; (ii) type 1_H effectively competes only against type 2_H, and thus c_H has an additive effect on p_H but does not affect the mark-up. Therefore suppliers' rents are independent of σ for $(\lambda, \sigma) \in \mathcal{C}$, and even though the suppliers' private informations on costs become more significant, under policy C the suppliers are unable to increase their rents. As we will see, this is not the case for the other information policies.

For a large σ, no symmetric pure strategy BNE exists, but a mixed strategy BNE exists since for each type i_k of supplier, as the payoff function in (2) is continuous in bids. However, characterizing such a BNE is not straightforward, and in Proposition 1(ii) we describe a mixed strategy BNE for $\lambda = \frac{1}{2}$ and a few

[11] This results holds because $(p_{ik} - c_k)[\lambda F(p_L - p) + (1 - \lambda)F(p_H - p)]$ (the demand function for the product of a supplier choosing price p) is log-concave in p, as we show in the proof of Proposition 1 in the appendix.

[12] Obviously, the first order conditions which deliver the prices in (7) fail to take into account this (non-local) deviation by type L. Notice also that we are considering the specific deviation in which type L bids $p_H - \theta$ only to fix the ideas, but it is conceivable that there exist even more profitable deviations for type L.

values of σ. In such a BNE each type H bids slightly less than $c_H + \frac{1}{2}\theta$, whereas each type L randomizes between two bids p_L^a, p_L^b which are both larger than $c_L + \frac{3}{2}\theta$. This makes the profit of each type L larger than when 1_L and 2_L play the pure strategy $c_L + \frac{3}{2}\theta$, and in particular p_L^b is relatively close to $p_H - \theta$, which implies that type L obtains in equilibrium the payoff from bidding close to $p_H - \theta$. Notice that in this BNE the winner may be selected inefficiently, since a type L loses with a positive probability against a type H.

3.2 Private revelation

Under the policy of private revelation, B privately (and truthfully) reveals q_i to supplier i, for $i = 1, 2$, that is before the auction is held. A strategy for supplier i is thus a function P_i which associates a bid to each pair $(c_i, q_i) \in \{c_L, c_H\} \times [\underline{q}, \bar{q}]$. Since suppliers are ex ante symmetric, we focus on symmetric BNE in which the function P_1 is identical to P_2, denoted with P. Precisely, a symmetric BNE in which all suppliers bid according to a function P is such that

$$\text{for any } c_i \in \{c_L, c_H\} \text{ and any } q_i \in [\underline{q}, \bar{q}], \quad \begin{array}{c} (p_i - c_i)\Pr\{q_i - p_i > q_j - P(c_j, q_j) \text{ for } j \neq i\} \\ \text{is maximized with respect to } p_i \text{ at } p_i = P(c_i, q_i) \end{array} \tag{8}$$

This setting with bidimensional private information turns out to be closely linked to a "standard auction" environment in which (i) an object is sold through a first price auction with two bidders; (ii) v_i is the valuation (privately observed) of bidder i for $i = 1, 2$; (iii) (v_1, v_2) are i.i.d., each with c.d.f. G. In such a setting a symmetric BNE is characterized by a bidding function β such that each bidder with type v_i bids $\beta(v_i)$, and it is well known that a unique symmetric BNE exists [see for instance Monteiro (2009)].

Proposition 2 below relies on this result to find the unique symmetric BNE under the PR policy and the key idea is quite simple. In (8), replace c_i using $v_i = q_i - c_i$ and then employ $s_i = q_i - p_i$, the score, as the choice variable of supplier i. Then the problem

$$\max_{p_i} \quad (p_i - c_i)\Pr\{q_i - p_i > q_j - P(c_j, q_j) \text{ for } j \neq i\} \tag{9}$$

can be written as

$$\max_{s_i} \quad (v_i - s_i)\Pr\{s_i > q_j - P(c_j, q_j) \text{ for } j \neq i\} \tag{10}$$

The formulation in (10) is useful since it is the decision problem of a bidder with valuation v_i in a standard auction, in which $q_j - P(c_j, q_j)$ is the bid of her opponent. Moreover, (10) reveals that the score s_i offered by supplier i depends only on v_i, and not on q_i and c_i separately.

Proposition 2 *Consider policy PR, and let G denote the common c.d.f. of v_1 and v_2. Then the unique symmetric BNE is such that*

$$P(c_i, q_i) = c_i + \int_{\underline{q}-c_H}^{q_i - c_i} \frac{G(y)}{G(q_i - c_i)} dy \qquad \text{for any } c_i, q_i \tag{11}$$

The supplier with the highest value wins and the buyer's expected payoff U_{PR}^B is $E[\min\{v_1, v_2\}]$, the expected second highest valuation.

We notice that (an analogous formulation of) Proposition 2 holds under much more general assumptions: we can allow for any number of suppliers and for any (i.i.d.) distributions of qualities and costs. We restrict to the particular setting described in Section 2 in order to compare PR with the other information policies.

The c.d.f. G of v_1 and v_2 is simple to derive, but we need to distinguish two cases. If $\sigma > \theta$, that is if $\omega > 1$, then the value of a supplier with cost c_L is certainly larger than the value of a supplier with cost c_H, and G is

$$G(v) = \begin{cases} (1-\lambda)\frac{v+c_H-\underline{q}}{\theta} & \text{if } v \in [\underline{q}-c_H, \bar{q}-c_H] \\ 1-\lambda & \text{if } v \in (\bar{q}-c_H, \underline{q}-c_L] \\ 1-\lambda+\lambda\frac{v+c_L-\underline{q}}{\theta} & \text{if } v \in (\underline{q}-c_L, \bar{q}-c_L] \end{cases}$$

such that G is constant for values between $\bar{q}-c_H$ and $\underline{q}-c_L$. In the opposite case of $\omega \le 1$, the set of possible values is the interval $[\underline{q}-c_H, \bar{q}-c_L]$ and G is[13]

$$G(v) = \begin{cases} (1-\lambda)\frac{v+c_H-\underline{q}}{\theta} & \text{if } v \in [\underline{q}-c_H, \underline{q}-c_L] \\ (1-\lambda)\frac{v+c_H-\underline{q}}{\theta}+\lambda\frac{v+c_L-\underline{q}}{\theta} & \text{if } v \in (\underline{q}-c_L, \bar{q}-c_H] \\ 1-\lambda+\lambda\frac{v+c_L-\underline{q}}{\theta} & \text{if } v \in (\bar{q}-c_H, \bar{q}-c_L] \end{cases}$$

Given G, it is straightforward to evaluate $E[\min\{v_1, v_2\}]$; we describe the results in next Corollary.

Corollary 1 *Under policy PR,*
(i) when $\omega \le 1$ the buyer's payoff U_{PR}^B is equal to $\underline{q} - c_L + \frac{1}{3}[1-(1-\lambda)(3+3\lambda\omega-\lambda\omega^2)\omega]\theta$ and the suppliers' rents U_{PR}^S are $\frac{1}{3}[1+2\lambda(1-\lambda)\omega^2(3-\omega)]\theta$;
(ii) when $\omega > 1$ the buyer's payoff is $\underline{q} - c_L + \frac{1}{3}[1+\lambda-\lambda^2-3(1-\lambda^2)\omega]\theta$ and the suppliers' rents are $[\frac{1}{3}(1-2\lambda+2\lambda^2)+2\lambda(1-\lambda)\omega]\theta$.

It is simple to see that an increase in ω above zero is harmful for B since it reduces (increases) the probability of high (low) values. More formally, the c.d.f. for each supplier's value when $\omega > 0$ is first order stochastically dominated by the c.d.f. when $\omega = 0$ and $E[\min\{v_1, v_2\}]$ is decreasing in ω. Conversely, as in C, an increase in ω increases the suppliers' ex ante expected rents.

3.3 Public revelation

Under the policy of Public revelation the buyer truthfully and publicly reveals the suppliers' qualities q_1, q_2 before the auction is held. Thus q_1, q_2 become commonly known, and without loss of generality we suppose that $q_2 \ge q_1$ and use $t \ge 0$ to denote the difference $q_2 - q_1$. Since each supplier privately observes the own cost, we use p_{iL}, p_{iH} to denote the bid of supplier i if she has cost c_L, and her bid if she has cost c_H, respectively. This is the same notation employed for policy C, but notice that under PU the common knowledge of q_1, q_2 generically puts the suppliers on an asymmetric footing since $q_1 \ne q_2$ except in a zero-measure set. We use 1_L and 1_H to denote the type of supplier 1 with cost c_L and the type with cost c_H, respectively; likewise, we use 2_L, 2_H to denote the two types of supplier 2.

In fact, this game is equivalent to a standard first price auction like the one described in Subsection 3.2, except that bidders have asymmetrically (and independently) distributed valuations such that the set of possible values for bidder 1 is $\{q_1 - c_H, q_1 - c_L\}$, the set of possible values for bidder 2 is $\{q_2 - c_H, q_2 - c_L\}$, and $\Pr\{v_1 = q_1 - c_L\} = \Pr\{v_2 = q_2 - c_L\} = \lambda$. We study this setting in Doni and Menicucci (2011), and indeed our Proposition 3, in which we describe the unique equilibrium outcome,[14] follows from Proposition 1 in Doni and Menicucci (2011).

[13] In case that $\omega = 1$, the middle interval collapses to one point.

[14] Multiple BNE exists because type 1_H (and type 1_L in one case) never wins in equilibrium, but equilibrium conditions require that she bids above c_H (above c_L) with probability one, and in a way that no type of supplier 2 has incentive to bid above $t + c_H$ (above $t + c_L$). Since there are many strategies of 1_H (of 1_L) which achieve this goal, multiple BNE exist, but this is not an issue since each BNE generates the same outcome in the sense that the winner and the payoff of each type of supplier and of B are the same across different BNE.

Notice that in Proposition 3(ii) an important role is played by two specific bids $\tilde{p}$ and $\hat{p}$, such that $\tilde{p}$ is the larger solution to the equation

$$(1-\lambda)p^2 - [(1-2\lambda)t + (1-\lambda)(c_H + c_L)]p + (1-\lambda)(t+c_H)c_L - \lambda t c_H = 0 \qquad (12)$$

and $\hat{p} \equiv (1-\lambda)\tilde{p} + \lambda(t + c_L)$. Precisely, satisfying (12) guarantees that the c.d.f. for the mixed strategy of supplier 1_L is continuous at $p = \tilde{p} - t$. Furthermore, $\hat{p}$ is such that the c.d.f. for the mixed strategy of supplier 2_L has value 0 at $p = \hat{p}$. In the proof of Proposition 3(ii) we show that $\tilde{p}$ satisfies $\max\{t+c_L, c_H\} < \tilde{p} < t + c_H$, and therefore $t + c_L < \tilde{p} < t + c_L + (1-\lambda)\sigma$.

Proposition 3 *Consider policy PU and suppose that B has revealed qualities q_1, q_2, with $t \equiv q_2 - q_1 > 0$. Then multiple BNE exist, but they all generate the same outcome as the following BNE.*
Type 1_H bids more than c_H with probability one and in such a way that no type of supplier 2 has incentive to bid above $t + c_H$; the bids of the other types depend on the parameters as follows.

(i) If $\lambda t \geq \sigma$, then types 2_L and 2_H bid $t + c_L$; type 1_L bids more than c_L with probability one and in such a way that no type of supplier 2 has incentive to bid above $t + c_L$.

(ii) If $\lambda t < \sigma$, then types 1_L,2_L,2_H play mixed strategies with support $[\hat{p} - t, c_H)$ for 1_L, $[\hat{p}, \tilde{p}]$ for 2_L, $[\tilde{p}, t + c_H]$ for 2_H, in which $\tilde{p}$ is the larger solution to (12) and $\hat{p} \equiv (1-\lambda)\tilde{p} + \lambda(t + c_L)$. The c.d.f. Φ_{1L}, Φ_{2L}, Φ_{2H} for the mixed strategies are

$$\Phi_{1L}(p_{1L}) = \begin{cases} \frac{p_{1L}+t-\hat{p}}{\lambda(p_{1L}+t-c_L)} & \text{for } p_{1L} \in [\hat{p} - t, \tilde{p} - t) \\ 1 - \frac{1-\lambda}{\lambda}\frac{c_H - p_{1L}}{p+t-c_H} & \text{for } p_{1L} \in [\tilde{p} - t, c_H) \end{cases}$$

$$\Phi_{2L}(p_{2L}) = 1 - \frac{1-\lambda}{\lambda}\frac{\tilde{p} - p_{2L}}{p_{2L} - c_L - t} \quad \text{for } p_{2L} \in [\hat{p}, \tilde{p}], \qquad \Phi_{2H}(p_{2H}) = \begin{cases} \frac{p_{2H} - \tilde{p}}{p_{2H} - c_L - t} & \text{for } p_{2H} \in [\tilde{p}, t + c_H) \\ 1 & \text{for } p_{2H} = t + c_H \end{cases}$$

and are such that Φ_{1L} is continuous at $p_{1L} = \tilde{p} - t$ and $\Phi_{2L}(\hat{p}) = 0$.

When $\lambda t \geq \sigma$, Proposition 3(i) establishes that each type of supplier 2 bids $t + c_L$ and wins against each type of supplier 1,[15] a quite intuitive result since a large t gives a large advantage to supplier 2 with respect to the differences in costs which may be determined by the suppliers' private signals. More in detail, (i) 1_L (1_H) will not offer to B a score higher than $q_1 - c_L$ (higher than $q_1 - c_H$); (ii) a bid of $t + c_L$ of supplier 2 is equivalent to offering a score of $q_2 - (t + c_L) = q_1 - c_L$, thus supplier 2 certainly wins if she bids $t + c_L$ (or perhaps slightly less); (iii) if q_2 is sufficiently larger than q_1 (i.e., if t is sufficiently large), then both 2_L and 2_H is willing to bid $t + c_L$ in order to earn t (for 2_L) or $t - \sigma$ (for 2_H). Precisely, when $\lambda t \geq \sigma$ type 2_H prefers winning for sure by bidding $t + c_L$ rather than bidding $t + c_H$ and winning only against type 1_H, that is with probability $1 - \lambda$.

On the other hand, if σ is large with respect to t (that is, if the advantage of 2 is small), then $\lambda t < \sigma$ holds and 2_H is less aggressive since she prefers to bid $t + c_H$ and win only against type 1_H rather than bidding $t + c_L$ and winning with certainty, as the latter alternative yields a low profit margin. Indeed, 2_H bids in the interval $[\tilde{p}, t + c_H]$, with $\tilde{p} > t + c_L$ (and with an atom at $t + c_H$), which means that she offers a score in $[q_1 - c_H, q_2 - \tilde{p}]$. The less aggressive bidding of 2_H allows 1_L to win with positive probability by bidding somewhat above c_L, and indeed the support for the equilibrium bids of 1_L is $[\hat{p} - t, c_H)$ with $\hat{p} - t > c_L$, which corresponds to a score in $(q_1 - c_H, q_2 - \hat{p}]$. As a consequence, also the lowest bid of 2_L is larger than $t + c_L$, as we see from Proposition 3(ii). Precisely, 2_L bids in the interval $[\hat{p}, \tilde{p}]$, which corresponds to a score in $[q_2 - \tilde{p}, q_2 - \hat{p}]$. Therefore, a large cost difference with respect to t generates a

[15] In a related setting, Maskin and Riley (2000) identify an analogous BNE and provide the intuition we describe here.

less aggressive behavior of all suppliers, compared to the case of a small value of σ, because 2_H is less aggressive and this induces also the other suppliers to be less aggressive.

Notice that, when $\lambda t < \sigma$, the highest valuation supplier does not always win. Precisely, let $\Pr\{1_L \text{ def } 2_L\}$ and $\Pr\{1_L \text{ def } 2_H\}$ denote the probability that 1_L wins against 2_L and the probability that 1_L wins against 2_H, respectively. Since 1_L offers a score in $(q_1 - c_H, q_2 - \hat{p}]$, 2_L offers a score in $[q_2 - \tilde{p}, q_2 - \hat{p}]$, and 2_H offers a score in $[q_1 - c_H, q_2 - \tilde{p}]$, it follows that $\Pr\{1_L \text{ def } 2_L\} > 0$ even though $q_2 - c_L > q_1 - c_L$ and $\Pr\{1_L \text{ def } 2_H\} \in (0,1)$ even though $q_2 - c_H \neq q_1 - c_L$.[16]

In next corollary we use $u^B_{PU}(q_1, q_2)$ to denote B's equilibrium payoff given q_1, q_2. From Proposition 3(i) it follows that $u^B_{PU}(q_1, q_2) = q_1 - c_L$ when $\lambda t \geq \sigma$, whereas if $\lambda t < \sigma$ we can evaluate $u^B_{PU}(q_1, q_2)$ as the difference between the social surplus (the expected value of the winner) and the suppliers' payoff.

Corollary 2 *Under policy PU, given q_1, q_2 such that $t = q_2 - q_1 > 0$, the buyer's payoff is*

$$u^B_{PU}(q_1, q_2) = \begin{cases} q_1 - c_L & \text{if } \lambda t \geq \sigma \\ q_1 - c_L + \lambda(2-\lambda)t - (1-\lambda^2)\sigma - \lambda\sqrt{[t + (1-\lambda)\sigma]^2 - 4\lambda(1-\lambda)t^2} & \\ -\lambda^2 t \Pr\{1_L \textit{ def } 2_L\} + \lambda(1-\lambda)(\sigma - t)\Pr\{1_L \textit{ def } 2_H\} & \text{if } \lambda t < \sigma \end{cases}$$

In the setting of GGD with $\sigma = 0$, given $q_2 > q_1$, supplier 2 wins by bidding $t + c_L$ and $u^B_{PU}(q_1, q_2) = q_1 - c_L$. As σ increases above 0, B's payoff is still $q_1 - c_L$ as long as the inequality $\lambda t \geq \sigma$ is satisfied, but if $\lambda t < \sigma$ then Proposition 3(ii) applies and each supplier offers a score between $q_1 - c_H$ and $q_2 - \hat{p}$. Since $\hat{p} > t + c_L$, it follows that $q_1 - c_H < u^B_{PU}(q_1, q_2) < q_1 - c_L$. Therefore a positive σ reduces B's payoff only in the states of the world such that $\lambda t < \sigma$. As we have explained above, a σ satisfying $\lambda t \leq \sigma$ induces 2_H to bid higher [i.e., less aggressively] than $t + c_L$, which in turn elicits less aggressive bidding also from 1_L and 2_L; this reduces B's payoff.

After the buyer's payoff $u^B_{PU}(q_1, q_2)$ is obtained for any q_1, q_2, we can derive B's ex ante expected payoff U^B_{PU} as $E_{q_1,q_2}[u^B_{PU}(q_1, q_2)]$. However, given that $\Pr\{1_L \text{ def } 2_L\}$ and $\Pr\{1_L \text{ def } 2_H\}$ have complicated expressions, also U^B_{PU} has a complicated expression. Nevertheless, in next section we show that in some cases it is possible to compare U^B_{PU} with U^B_{PR} without resorting to numerical methods.

4 The optimal information policy

In this section we compare the three information revelation policies from the point of view of B.

4.1 Comparison between PR and C

Proposition 4 *(i) If $\sigma \leq \theta$, then $U^B_{PR} > U^B_C$ for any $\lambda \in (0,1)$.*
(ii) If $(\lambda, \sigma) \in \mathcal{C}$ and $\sigma > \frac{2\lambda^2 - 14\lambda + 13}{12\lambda(1-\lambda)}\theta$, then $U^B_C > U^B_{PR}$.

Given that $U^B_{PR} > U^B_C$ when $\sigma = 0$, Proposition 4(i) can be interpreted as an extension of the result obtained by GGD. On the other hand, when $\sigma > \frac{2\lambda^2 - 14\lambda + 13}{12\lambda(1-\lambda)}\theta$ and $(\lambda, \sigma) \in \mathcal{C}$ we find that $U^B_C > U^B_{PR}$, and a simple intuition applies to this result. In Subsections 3.1 we have seen that under C a large σ separates type L from type H in the sense that type L always wins against type H. This implies that the winner is always the supplier with the highest valuation, and therefore C and PR generate the same social surplus. Thus the buyer's preferences between C and PR are determined by the suppliers' payoffs under the two policies, U^S_C and U^S_{PR}. In this respect, we have noticed in Subsection 3.1 that U^S_C is constant with respect

[16] The precise values of $\Pr\{1_L \text{ def } 2_L\}$ and $\Pr\{1_L \text{ def } 2_H\}$ are obtained in the proof of Proposition 3.

to σ if $(\lambda, \sigma) \in \mathcal{C}$, and from Corollary 1(ii) it follows that U^S_{PR} is increasing in σ. This suggests that C is superior to PR for a large σ because is more effective at controlling the suppliers' rents.

4.2 Comparison between PR and PU

In Subsection 3.3 (Corollary 2) we have obtained the buyer's payoff under PU for given values of q_1, q_2 such that $q_2 \geq q_1$, $u^B_{PU}(q_1, q_2)$, and noticed that B's ex ante payoff U^B_{PU} is equal to $E_{q_1,q_2}[u^B_{PU}(q_1, q_2)]$. In order to compare PU with PR, we recall that B's payoff under PR is given by the expected second highest value: $U^B_{PR} = E_{v_1,v_2}[\min\{v_1, v_2\}] = E_{q_1,c_1,q_2,c_2}[\min\{q_1 - c_1, q_2 - c_2\}]$, and we define $u^B_{PR}(q_1, q_2)$ as $E_{c_1,c_2}[\min\{q_1 - c_1, q_2 - c_2\}|q_1, q_2]$ in order to satisfy $U^B_{PR} = E_{q_1,q_2}[u^B_{PR}(q_1, q_2)]$. Thus U^B_{PR} can be obtained by first evaluating $u^B_{PR}(q_1, q_2)$ for each q_1, q_2, and then taking the expectation of $u^B_{PR}(q_1, q_2)$ with respect to q_1, q_2. Since $U^B_{PU} = E_{q_1,q_2}[u^B_{PU}(q_1, q_2)]$, this suggests that comparing $u^B_{PR}(q_1, q_2)$ with $u^B_{PU}(q_1, q_2)$ for different (q_1, q_2) may give some insights about the comparison between U^B_{PR} and U^B_{PU}.

To this purpose it is useful to recall our earlier remark that the first score auction under PU is equivalent to a standard first price auction (FPA henceforth) in which the set of possible values for bidder 1 is $\{q_1 - c_H, q_1 - c_L\}$, the set of possible values for bidder 2 is $\{q_2 - c_H, q_2 - c_L\}$, and v_1, v_2 are independently distributed with $\Pr\{v_1 = q_1 - c_L\} = \Pr\{v_2 = q_2 - c_L\} = \lambda$. Conversely, for PR we have $u^B_{PR}(q_1, q_2) = E_{c_1,c_2}[\min\{q_1 - c_1, q_2 - c_2\}|q_1, q_2]$, which is a seller's expected revenue in a second price auction (SPA henceforth) given the same information environment described just above. Therefore, comparing $u^B_{PR}(q_1, q_2)$ with $u^B_{PU}(q_1, q_2)$ leads us to the long standing problem in auction theory of comparing the seller's expected revenue in the FPA with the expected revenue in the SPA when the bidders' values are asymmetrically distributed.

In particular, in our environment the asymmetry is generated by a distribution shift with size $q_2 - q_1$. The literature on asymmetric auctions has studied a related setting: Maskin and Riley (2000) and Kirkegaard (2011) prove that with continuously distributed valuations (and some regularity conditions) the FPA is superior to the SPA. However, we consider a model with discretely distributed valuations, and the above results do not necessarily apply. In particular, we obtain an opposite result in the case of a small shift.

In order to compare $u^B_{PU}(q_1, q_2)$ with $u^B_{PR}(q_1, q_2)$, it is useful to define three subsets of $Q = [\underline{q}, \bar{q}] \times [\underline{q}, \bar{q}]$, which we denote Q_1, Q_2, Q_3:

$$
\begin{aligned}
Q_1 &\equiv \{(q_1, q_2) \in Q : \quad q_1 \leq q_2 < q_1 + \sigma\} \\
Q_2 &\equiv \{(q_1, q_2) \in Q : \quad q_1 + \sigma \leq q_2 < q_1 + \frac{\sigma}{\lambda}\} \\
Q_3 &\equiv \{(q_1, q_2) \in Q : \quad q_1 + \frac{\sigma}{\lambda} \leq q_2\}
\end{aligned}
$$

Clearly, Q_3 is empty if $\frac{\sigma}{\lambda} > \theta$ and Q_2 is empty if $\sigma > \theta$. Figure 1 represents graphically Q_1, Q_2, Q_3

Insert figure 1 here

The buyer's payoff depends as follows on the set in which (q_1, q_2) is located.

- Under PU, if $(q_1, q_2) \in Q_3$ then Proposition 3(i) applies and $u^B_{PU}(q_1, q_2) = q_1 - c_L$. Conversely, Proposition 3(ii) applies if $(q_1, q_2) \in Q_1 \cup Q_2$ and then $q_1 - c_H < u^B_{PU}(q_1, q_2) < q_1 - c_L$.

- Under PR, for each $(q_1, q_2) \in Q_2 \cup Q_3$ we have $q_2 - q_1 \geq \sigma$, thus $q_1 - c_1 < q_2 - c_2$. This implies that $u^B_{PR}(q_1, q_2) = E_{c_1,c_2}[\min\{q_1 - c_1, q_2 - c_2\}|q_1, q_2] = E_{c_1}(q_1 - c_1|q_1) = q_1 - c_L - (1-\lambda)\sigma$. Conversely, for $(q_1, q_2) \in Q_1$ we find $u^B_{PR}(q_1, q_2) = \lambda^2(q_1 - c_L) + \lambda(1-\lambda)(q_1 + t - c_L - \sigma) + \lambda(1-\lambda)(q_1 - c_L - \sigma) + (1-\lambda)^2(q_1 - c_L - \sigma) = q_1 - c_L - (1-\lambda)\sigma - \lambda(1-\lambda)(\sigma - t)$, which is smaller than $q_1 - c_L - (1-\lambda)\sigma$.

Next proposition relies on the above arguments to compare U^B_{PR} with U^B_{PU}, and we remark that it does not use the assumption that (q_1, q_2) are uniformly distributed.

Proposition 5 *Suppose that q_1, q_2 are i.i.d. with common support $[\underline{q}, \bar{q}]$. Then*
(i) $U^B_{PU} > U^B_{PR}$ when $\sigma > 0$ is about zero;
(ii) $U^B_{PR} > U^B_{PU}$ when $\sigma \geq \max\{\frac{3(1+\lambda)}{2(3-\lambda)}, \frac{3\lambda-1}{2(1-\lambda)}\}\theta$.

The basic idea for Proposition 5(i) is quite simple. When $\sigma > 0$ is small, $\Pr\{(q_1, q_2) \in Q_1 \cup Q_2\}$ is about 0, which suggests that the comparison between PU and PR is determined by the comparison between $u^B_{PU}(q_1, q_2)$ and $u^B_{PR}(q_1, q_2)$ for $(q_1, q_2) \in Q_3$. Then (i) under PU each type of supplier 2 offers a score $q_1 - c_L$ in order to win against both types of supplier 1; (ii) under PR the second highest value is $v_1 = q_1 - c_1$, which in expectation is equal to $q_1 - c_L - (1-\lambda)\sigma$, smaller than $q_1 - c_L$. This is the "Getty effect" described by Maskin and Riley (2000), which applies when the size of the shift is large relative to σ. Although this argument may appear sufficient to establish $U^B_{PU} > U^B_{PR}$, it is necessary to notice that $u^B_{PU}(q_1, q_2) - u^B_{PR}(q_1, q_2) = (1-\lambda)\sigma$ when $(q_1, q_2) \in Q_3$, and thus the advantage of PU over PR is small when σ is small. However, in the proof of Proposition 5(i) we show that PR cannot be much better than PU in $Q_1 \cup Q_2$, as $u^B_{PU}(q_1, q_2) - u^B_{PR}(q_1, q_2) > -\lambda\sigma$ for any $(q_1, q_2) \in Q_1 \cup Q_2$, and then the property that $\Pr\{(q_1, q_2) \in Q_1 \cup Q_2\}$ is close to 0 for σ close to zero yields $U^B_{PU} > U^B_{PR}$.

Proposition 5(ii) relies on a different argument. First notice that $\max\{\frac{3(1+\lambda)}{2(3-\lambda)}, \frac{3\lambda-1}{2(1-\lambda)}\} > \lambda$, thus Q_3 is empty when $\sigma \geq \max\{\frac{3(1+\lambda)}{2(3-\lambda)}, \frac{3\lambda-1}{2(1-\lambda)}\}\theta$, and any feasible (q_1, q_2) (such that $q_2 \geq q_1$) belongs to $Q_1 \cup Q_2$, which makes Proposition 3(ii) apply. Then we show that for any $(q_1, q_2) \in Q_1 \cup Q_2$, the suppliers' aggregate rents are lower under PR than under PU given $\sigma \geq \max\{\frac{3(1+\lambda)}{2(3-\lambda)}, \frac{3\lambda-1}{2(1-\lambda)}\}\theta$. Moreover, we know from Proposition 2 that social surplus is maximized under PR since the higher value supplier always wins under PR. Conversely, under PU social surplus is not maximized as the lowest valuation supplier may win with positive probability. Therefore it follows that $U^B_{PR} > U^B_{PU}$ if $\sigma \geq \max\{\frac{3(1+\lambda)}{2(3-\lambda)}, \frac{3\lambda-1}{2(1-\lambda)}\}\theta$.

In fact, there is another interesting way of seeing this result, which relies on inquiring how a small shift $q_2 - q_1 > 0$ affects the suppliers' bids with respect to the case of $q_2 = q_1$. We find that type 1_L bids less aggressively while the bidding of types 2_L and 2_H in terms of score is unchanged (up to a second order effect). This has the consequence that the buyer's payoff under PU decreases as a consequence of the shift, that is $u^B_{PU}(q_1, q_2) < u^B_{PU}(q_1, q_1)$ [see Doni and Menicucci (2011) for more details on this result]. On the other hand, the buyer's payoff under PR is the expected second highest valuation, and thus $u^B_{PR}(q_1, q_2) > u^B_{PR}(q_1, q_1)$, which suggests that PR is superior to PU for a small shift, or equivalently for a large σ/θ.

The main message of Proposition 5 is as follows. When $\sigma > 0$ is small, revealing q_1, q_2 typically puts the bidders in a very asymmetric setting such that $\lambda t \geq \sigma$ is satisfied, and then the supplier with the highest quality bids aggressively in order to win against each type of the other supplier. This makes PU superior to PR. When instead σ is large such that $\lambda t < \sigma$ holds in any case (i.e., for any t), no supplier has a sufficiently large advantage to bid aggressively in PU (in fact, a modest asymmetry reduces the buyer's payoff under PU), and then we find PR is better.

4.3 General ranking

By using Propositions 4 and 5 we obtain next Proposition. It does not cover all parameter values, but shows that for each fixed policy there is a set of parameter values for which the given policy is optimal.

Proposition 6 *(i) The best information revelation policy is PU if σ is about zero; PR if* $\max\{\frac{3(1+\lambda)}{2(3-\lambda)}, \frac{3\lambda-1}{2(1-\lambda)}\}\theta \leq \sigma \leq \theta$*;C if* $(\lambda, \sigma) \in \mathcal{C}$ *and* $\sigma \geq \max\{\frac{2\lambda^2-14\lambda+13}{12\lambda(1-\lambda)}, \frac{3\lambda-1}{2(1-\lambda)}\}\theta$.

(ii) For the special case in which $\lambda = \frac{1}{2}$*, there exists* $\omega^* \simeq 0.3949$ *such that the best information revelation policy is PU if* $\omega < \omega^*$*, is PR if* $\omega^* < \omega < \frac{13}{6}$*, is C if* $\omega \in (\frac{13}{6}, 2\sqrt{3}-1] \cup \{\frac{5}{2}, 3, \frac{7}{2}\}$.

Under the assumptions of Proposition 6(ii), the analysis of GGD reveals that $\omega = 0$ implies $U^B_{PR} = U^B_{PU} > U^B_C$. Proposition 6(ii) establishes that these results are not robust to allowing $\omega > 0$: $U^B_{PU} \neq U^B_{PR}$ for any $\omega \neq \omega^*$, and C is the best policy for large values of ω. More generally, Proposition 6(ii) implies that the amount of information B should release is decreasing with respect to ω. Precisely, revealing q_1 and q_2 is the best policy for B when each supplier's private information on costs is scarcely significant (since c_L is relatively close to c_H when ω is small), but private revelation is optimal when ω takes on intermediate values, and no revelation at all is optimal when ω is large, that is when suppliers' private information is very significant with respect to quality differences. As we have seen in the previous subsections, PU is optimal for $\omega \simeq 0$ as it elicits aggressive bidding from the supplier with higher quality, whereas C is optimal for a large ω since it is good at controlling rents.

5 Conclusions

In some procurement settings a product's perceived quality depends on the buyer's subjective needs or tastes. In these cases buyers have to choose how much of their information to reveal to suppliers before the bidding occurs. In this paper we have studied the outcomes of different information revelation policies in a procurement auction. GGD have already delved into this issue, under the assumption that suppliers have identical, commonly known costs. We introduce uncertainty in each suppliers' cost, and therefore suppliers can be heterogeneous both because of the qualities of their products and because of their costs. Under uniformly distributed qualities, our main finding is (Proposition 6) that the amount of information the buyer should reveal is decreasing in the degree of uncertainty on suppliers' costs with respect to the degree of uncertainty on supplier's qualities. In order to derive this result we compared the revenue in a FPA with the revenue in a SPA in an asymmetric setting with a distribution shift, and we have proved that some known results for asymmetric auctions with continuos valuations do not hold in an environment with discretely distributed types.

Future research could extend our analysis along several directions. For instance, we could allow for an endogenous number of suppliers, which requires to analyze how the information policy affects each supplier's entry incentives. Another extension would allow for vertically differentiated products. Precisely, sometimes the qualities of two different products can be unambiguously ranked, and the uncertainty is only about the buyer's willingness to pay for the higher quality product rather than for the lower quality one. This suggests the question of how different information policies affect a supplier's incentives to invest in improving the quality of the own product when vertical differentiation is endogenous. Finally, the evaluation of a procurement policy should take into account its vulnerability to corruption and collusion [see Katok and Wambach (2008)]. Therefore, it would be useful to have an analysis of pros and cons of different procurement policies when qualitative evaluations are expressed by public officials who can pursue their private interest, or when suppliers can create a cartel.

6 Appendix

6.1 Proof of Proposition 1

Without loss of generality, we consider supplier 1. Given the bids p_L, p_H of the two types of supplier 2, the payoff of type L of supplier 1 is $(p_{1L} - c_L)[\lambda F(p_L - p_{1L}) + (1-\lambda)F(p_H - p_{1L})]$, and the payoff of type H of supplier 1 is $(p_{1H} - c_H)[\lambda F(p_L - p_{1H}) + (1-\lambda)F(p_H - p_{1H})]$. In equilibrium these payoffs are maximized at $p_{1L} = p_L$ and at $p_{1H} = p_H$, respectively. The associated first order conditions are written as follows, with $\Delta = p_H - p_L$ (clearly, $\Delta \geq 0$ since $c_H > c_L$):[17]

$$\begin{aligned} \lambda F(0) + (1-\lambda)F(\Delta) &= (p_L - c_L)[\lambda f(0) + (1-\lambda)f(\Delta)] && (13) \\ \lambda F(-\Delta) + (1-\lambda)F(0) &= (p_H - c_H)[\lambda f(-\Delta) + (1-\lambda)f(0)] && (14) \end{aligned}$$

As it is intuitive, we need to distinguish the case in which $\Delta \leq \theta$ from the case in which $\Delta > \theta$, since $F(-\Delta) = 0$, $F(\Delta) = 1$ and $f(-\Delta) = f(\Delta) = 0$ when $\Delta > \theta$, whereas $0 < F(-\Delta) < F(\Delta) < 1$ and $f(-\Delta) = f(\Delta) > 0$ when $\Delta < \theta$.

6.1.1 The case in which $\Delta < \theta$

Step 1 Derivation of equilibrium bids and the payoff of the buyer When $\Delta \leq \theta$, using (1) we can write (13) and (14) as follows:

$$\begin{aligned} \frac{1}{2}\theta + (1-\lambda)\Delta(1 - \frac{\Delta}{2\theta}) &= (p_L - c_L)[1 - (1-\lambda)\frac{\Delta}{\theta}] && (15) \\ \frac{1}{2}\theta - \lambda\Delta(1 - \frac{\Delta}{2\theta}) &= (p_H - c_H)(1 - \frac{\lambda\Delta}{\theta}) && (16) \end{aligned}$$

and taking the difference between (15) and (16) yields

$$\frac{1}{2}\Delta(2\theta - \Delta) = (\theta - \Delta + \lambda\Delta)(\sigma - \Delta) + (p_H - c_H)(2\lambda - 1)\Delta \tag{17}$$

If $\lambda = \frac{1}{2}$, then (17) has solutions $\Delta = \frac{1}{2}\sigma$ and $\Delta = 2\theta$. The second solution violates $\Delta \leq \theta$, whereas inserting $\Delta = \frac{1}{2}\sigma$ in (15)-(16) we obtain (4).

If $\lambda \neq \frac{1}{2}$, then from (17) we obtain $p_H = c_H + \frac{(3-2\lambda)\Delta^2 - 2(2\theta + (1-\lambda)\sigma)\Delta + 2\theta\sigma}{2(1-2\lambda)\Delta}$ and $p_H - \Delta$ is equal to p_L in (5). Inserting the expression for p_H into (16) yields (6), in which Δ is the unique unknown; we are interested in the solutions of (6) which belong to $[0, \theta]$. It is useful to divide the left hand side in (6) by θ^3, with $x \equiv \frac{\Delta}{\theta}$, $\omega \equiv \frac{\sigma}{\theta}$, and then (6) is written as

$$4\lambda(1-\lambda)x^3 - (3 + 4\lambda(1-\lambda) + 2\lambda(1-\lambda)\omega)x^2 + (5 - 2\lambda + 2\omega)x - 2\omega = 0 \tag{18}$$

There exists a $\Delta \in [0, \theta]$ which solves (6) if and only if there exists $x \in [0, 1]$ which solves (18). We use $\Psi(x)$ to denote the left hand side of (18) and notice that $\Psi(0) = -2\omega < 0$, $\Psi(1) = 2(1-\lambda)(1-\lambda\omega)$; thus there exists a solution in $(0, 1]$ as long as $\omega \leq \frac{1}{\lambda}$, which is equivalent to $\lambda\sigma \leq \theta$. Moreover, $\Psi''(x) = 24\lambda(1-\lambda)x - 2(3 + 4\lambda(1-\lambda) + 2\lambda(1-\lambda)\omega)$ is negative for any $x \in (0, 1)$, thus Ψ is strictly concave in $[0, 1]$ and the solution to (18) in $(0, 1]$ is unique for $\omega < \frac{1}{\lambda}$. Precisely, let x^* denote the smallest solution to (18) in $(0, 1)$ and notice that the strict concavity of Ψ and $\Psi(1) > 0$ imply $\Psi(x) > 0$ for each $x \in (x^*, 1]$. Hence, (6) has a unique solution in $(0, \theta]$ if $\lambda\sigma < \theta$.

[17] Notice that the payoff functions are continuously differentiable in prices.

Given the equilibrium prices in (4)-(5), we find that the payoff of the buyer is

$$\begin{aligned} U_C^B &= \lambda^2(E[\max\{q_1,q_2\}]-p_L)+(1-\lambda)^2(E[\max\{q_1,q_2\}]-p_L-\Delta) \\ &\quad +2\lambda(1-\lambda)\Big(\int_{\underline{q}}^{\underline{q}+\theta-\Delta}\int_{q_1+\Delta}^{\underline{q}+\theta}(q_2-p_L-\Delta)\frac{1}{\theta^2}dq_2dq_1+\int_{\underline{q}}^{\underline{q}+\theta}\int_{\underline{q}}^{\min\{q_1+\Delta,\underline{q}+\theta\}}(q_1-p_L)\frac{1}{\theta^2}dq_2dq_1\Big) \\ &= \underline{q}-p_L+\frac{2}{3}\theta-(1-\lambda)\,\Delta\frac{3\theta^2-3\lambda\Delta\theta+\lambda\Delta^2}{3\theta^2} \end{aligned}$$

Step 2 Proof that p_L, p_H in (3) constitute a BNE if $\lambda\sigma<\theta$ Clearly, merely satisfying the first order conditions does not guarantee that a BNE is obtained. In order to verify that this is the case, consider the function $R(y)\equiv\frac{\lambda f(y)+(1-\lambda)f(\Delta+y)}{\lambda F(y)+(1-\lambda)F(\Delta+y)}$ defined for $y\in(-\theta-\Delta,\theta]$. If R is monotone decreasing in y, then it is straightforward to prove that p_L, p_H in (3) constitute a BNE. Next lemma establishes that R is strictly decreasing when $\lambda\le\frac{1}{2}$, but not necessarily when $\lambda>\frac{1}{2}$.

Step 2.1 The function $R(y)=\frac{\lambda f(y)+(1-\lambda)f(\Delta+y)}{\lambda F(y)+(1-\lambda)F(\Delta+y)}$ is strictly decreasing in y in the interval $(-\theta-\Delta,\theta]$ if $\lambda\le\frac{1}{2}$; if $\lambda>\frac{1}{2}$, then R is strictly decreasing in the interval $[-\Delta,\theta]$ We start by exploiting (1) to derive an expression for $R(y)$:

$$R(y)=\begin{cases} \frac{2}{\theta+\Delta+y} & \text{for } y\in(-\theta-\Delta,-\theta] \\ \frac{2(\theta+\Delta+y-\lambda\Delta)}{y^2+2(\Delta+\theta-\lambda\Delta)y+\theta^2+2\theta\Delta-2\lambda\theta\Delta+\Delta^2-\lambda\Delta^2} & \text{for } y\in(-\theta,-\Delta] \\ \frac{2[(2\lambda-1)y+\lambda\Delta+\theta-\Delta]}{(2\lambda-1)y^2+2(\theta-\Delta+\lambda\Delta)y-\Delta^2+\theta^2+2\theta\Delta-2\lambda\theta\Delta+\lambda\Delta^2} & \text{for } y\in(-\Delta,0] \\ \frac{2(\theta-\Delta-y+\lambda\Delta)}{-y^2+2(\theta-\Delta+\lambda\Delta)y+\theta^2+2\theta\Delta-2\lambda\theta\Delta-\Delta^2+\lambda\Delta^2} & \text{for } y\in(0,\theta-\Delta] \\ \frac{2\lambda(y-\theta)}{\lambda\theta^2-2\lambda\theta y+\lambda y^2-2\theta^2} & \text{for } y\in(\theta-\Delta,\theta] \end{cases}$$

- For $y\in(-\theta-\Delta,-\theta]$ it is straightforward that R is decreasing in y.
- For $y\in(-\theta,-\Delta]$, $R'(y)$ has the same sign as $\rho_1(y)\equiv-y^2-2(\Delta+\theta-\lambda\Delta)y-\theta^2-2(1-\lambda)\Delta\theta+(2\lambda-1)(1-\lambda)\Delta^2$, which is decreasing in y and $\rho_1(-\theta)=-(1-2\lambda)(1-\lambda)\Delta^2\le0$ for $\lambda\le\frac{1}{2}$.
- For $y\in(-\Delta,0]$, $R'(y)$ has the same sign as

  $\rho_2(y)\equiv-(2\lambda-1)^2y^2-2(2\lambda-1)(\theta-\Delta+\lambda\Delta)y-(3-2\lambda)\theta^2+2(2\lambda+1)(1-\lambda)\Delta\theta-(1-\lambda)\Delta^2$

  Case of $\lambda\le\frac{1}{2}$. Then ρ_2 is increasing and $\rho_2(0)=-(3-2\lambda)\theta^2+2(2\lambda+1)(1-\lambda)\Delta\theta-(1-\lambda)\Delta^2$, which is decreasing in θ. Hence, from $\theta>\Delta$ we see that $\rho_2(0)$ is smaller than $-(3-2\lambda)\Delta^2+2(2\lambda+1)(1-\lambda)\Delta^2-(1-\lambda)\Delta^2=-(2-5\lambda+4\lambda^2)\Delta^2<0$.

  Case of $\lambda>\frac{1}{2}$. Then ρ_2 is decreasing and $\rho_2(-\Delta)=-(3-2\lambda)\theta^2+2\lambda(3-2\lambda)\Delta\theta-\lambda\Delta^2$, which is decreasing in θ. Hence, from $\theta>\Delta$ we see that $\rho_2(-\Delta)<-(3-2\lambda)\Delta^2+2\lambda(3-2\lambda)\Delta^2-\lambda\Delta^2=(4\lambda-3)(1-\lambda)\Delta^2$, which is negative or zero for $\lambda\le\frac{3}{4}$. For $\lambda>\frac{3}{4}$, notice that $\theta\ge\frac{\Delta}{2-\frac{4}{3}\lambda}$ since $\Psi(2-\frac{4}{3}\lambda)=\frac{2}{9}(4\lambda^2-6\lambda+3)(4\lambda^2-10\lambda+3)\omega+\frac{2}{27}(2\lambda-3)(64\lambda^4-208\lambda^3+216\lambda^2-90\lambda+9)$ is non negative for each $\omega\in[0,\frac{1}{\lambda}]$, and thus $\rho_2(-\Delta)<-(3-2\lambda)(\frac{\Delta}{2-\frac{4}{3}\lambda})^2+2\lambda(3-2\lambda)\Delta\frac{\Delta}{2-\frac{4}{3}\lambda}-\lambda\Delta^2=-\frac{(4\lambda-3)^2}{4(3-2\lambda)}\Delta^2<0$.
- For $y\in(0,\theta-\Delta]$, $R'(y)$ has the same sign as $\rho_3(y)\equiv-y^2+2(\theta-\Delta+\lambda\Delta)y-3\theta^2+2(1-\lambda)\Delta\theta+(2\lambda-1)(1-\lambda)\Delta^2$, which is increasing in y and $\rho_3(\theta-\Delta)=-2\theta^2+\lambda\Delta^2-2\lambda^2\Delta^2\le-(2-\lambda+2\lambda^2)\Delta^2<0$.
- For $y\in(\theta-\Delta,\theta]$, $R'(y)$ has the same sign as $\rho_4(y)\equiv-\lambda y^2+2\lambda\theta y-\lambda\theta^2-2\theta^2$, which is increasing in y and $\rho_4(\theta)=-2\theta^2<0$.

Step 2.2 If type L and type H of supplier 2 play p_L and p_H in (3), respectively, then playing p_H is a best reply for type H of supplier 1 As we have specified above, the payoff function of type H is $\pi_H(p_{1H}) = (p_{1H} - c_H)[\lambda F(p_L - p_{1H}) + (1-\lambda)F(p_H - p_{1H})]$, and

$$\pi'_H(p_{1H}) = [\lambda f(p_L - p_{1H}) + (1-\lambda)f(p_H - p_{1H})][\frac{\lambda F(p_L - p_{1H}) + (1-\lambda)F(p_H - p_{1H})}{\lambda f(p_L - p_{1H}) + (1-\lambda)f(p_H - p_{1H})} - p_{1H} + c_H]$$

We know that $\pi'_H(p_H) = 0$ and Step 2.1 reveals that if $\lambda \leq \frac{1}{2}$, then the second factor in $\pi'_H(p_{1H})$ is strictly decreasing in p_{1H} – interpret $p_L - p_{1H}$ as y – for $p_{1H} \in [p_L - \theta, p_H + \theta)$. Therefore $\pi'_H(p_H) = 0$ implies that $\pi'_H(p_{1H}) > 0$ for $p_{1H} \in [p_L - \theta, p_H)$ and $\pi'_H(p_{1H}) < 0$ for $p_{1H} \in (p_H, p_H + \theta)$. If instead $\lambda > \frac{1}{2}$, then the second factor in $\pi'_H(p_{1H})$ is strictly decreasing in p_{1H} for $p_{1H} \in [p_L - \theta, p_H)$. For $p_{1H} \in (p_H, p_L + \theta]$ we obtain

$$\begin{aligned}\pi'_H(p_{1H}) \quad = \quad & \frac{1}{2} + \frac{\lambda}{\theta}(p_L - p_{1H} + \frac{1}{2\theta}(p_L - p_{1H})^2) + \frac{1-\lambda}{\theta}(p_H - p_{1H} + \frac{1}{2\theta}(p_H - p_{1H})^2) + \\ & - \frac{p_{1H} - c_H}{\theta}(1 + \frac{\lambda}{\theta}(p_L - p_{1H}) + \frac{1-\lambda}{\theta}(p_H - p_{1H}))\end{aligned}$$

Since π'_H is convex and $\pi'_H(p_H) = 0$, the inequality $\pi'_H(p_{1H}) \leq 0$ holds for each $p_{1H} \in (p_H, p_L + \theta]$ if and only if $\pi'_H(p_L + \theta) \leq 0$. We find that $\pi'_H(p_L + \theta) = \frac{1-\lambda}{2\lambda-1}[2\lambda\hat{x}^2 - (1 - \lambda\omega + \omega + 2\lambda)\hat{x} + \omega]$, where $\hat{x}$ is the unique solution to $\Psi(x) = 0$ in $(0,1)$ [see (18)].

Step 2.2.1 For $\omega < 1$, the inequality $\pi'_H(p_L + \theta) \leq 0$ is satisfied First we prove that $\frac{1}{2}\omega < \hat{x} < \frac{3}{4}\omega$, given $\lambda > \frac{1}{2}$ and $\omega < 1$, by verifying that $\Psi(\frac{1}{2}\omega) < 0 < \Psi(\frac{3}{4}\omega)$. Precisely, we find that $\Psi(\frac{1}{2}\omega) = \frac{1}{4}\omega\,(2\lambda - 1)\,(2\lambda\omega - \omega - 2) < 0$ and $\Psi(\frac{3}{4}\omega) = \frac{1}{16}\omega(9\omega\,(4-\omega)\,\lambda^2 + \left(9\omega^2 - 36\omega - 24\right)\lambda + 28 - 3\omega)$. Let $\xi_\omega(\lambda) = 9\omega\,(4-\omega)\,\lambda^2 + \left(9\omega^2 - 36\omega - 24\right)\lambda + 28 - 3\omega$. For $\omega \leq 2 - \frac{2}{3}\sqrt{3}$, we find that ξ_ω is maximized with respect to $\lambda \in (\frac{1}{2}, 1]$ at $\lambda = 1$and $\xi_\omega(1) = 9\omega\,(4-\omega) + \left(9\omega^2 - 36\omega - 24\right) + 28 - 3\omega = 4 - 3\omega > 0$; for $\omega \in (2 - \frac{2}{3}\sqrt{3}, 1]$, we find that ξ_ω is maximized with respect to $\lambda \in (\frac{1}{2}, 1]$ at $\lambda = \frac{8+12\omega-3\omega^2}{6\omega(4-\omega)}$, with $\xi_\omega(\frac{8+12\omega-3\omega^2}{6\omega(4-\omega)}) = 9\omega\,(4-\omega)\,(\frac{8+12\omega-3\omega^2}{6\omega(4-\omega)})^2 + \left(9\omega^2 - 36\omega - 24\right)\frac{8+12\omega-3\omega^2}{6\omega(4-\omega)} + 28 - 3\omega = \frac{-9\omega^4+84\omega^3-256\omega^2+256\omega-64}{4\omega(4-\omega)}$, which is positive for $\omega \in (2 - \frac{2}{3}\sqrt{3}, 1]$.

Finally, at $x = \frac{1}{2}\omega$ we find $\pi'_H(p_L+\theta) = \frac{1-\lambda}{2\lambda-1}(2\lambda(\frac{1}{2}\omega)^2 - (1-\lambda\omega+\omega+2\lambda)\frac{1}{2}\omega + \omega) = -\frac{1}{2}\,(1-\omega)\,\omega\,(1-\lambda) \leq 0$; at $x = \frac{3}{4}\omega$ we find $\pi'_H(p_L+\theta) = \frac{1-\lambda}{2\lambda-1}(2\lambda(\frac{3}{4}\omega)^2 - (1-\lambda\omega+\omega+2\lambda)\frac{3}{4}\omega + \omega) = \frac{\omega(1-\lambda)}{8(2\lambda-1)}(15\lambda\omega + 2 - 6\omega - 12\lambda)$. max wrto λ: $\lambda = \frac{1}{2}$ if $\omega < \frac{4}{5}$, $\lambda = 1$ if $\omega > \frac{4}{5}$. In the first case, $15 \cdot \frac{1}{2}\omega + 2 - 6\omega - 12 \cdot \frac{1}{2} = \frac{3}{2}\omega - 4 < 0$; in the second case, $15\omega + 2 - 6\omega - 12 = 9\omega - 10 < 0$.

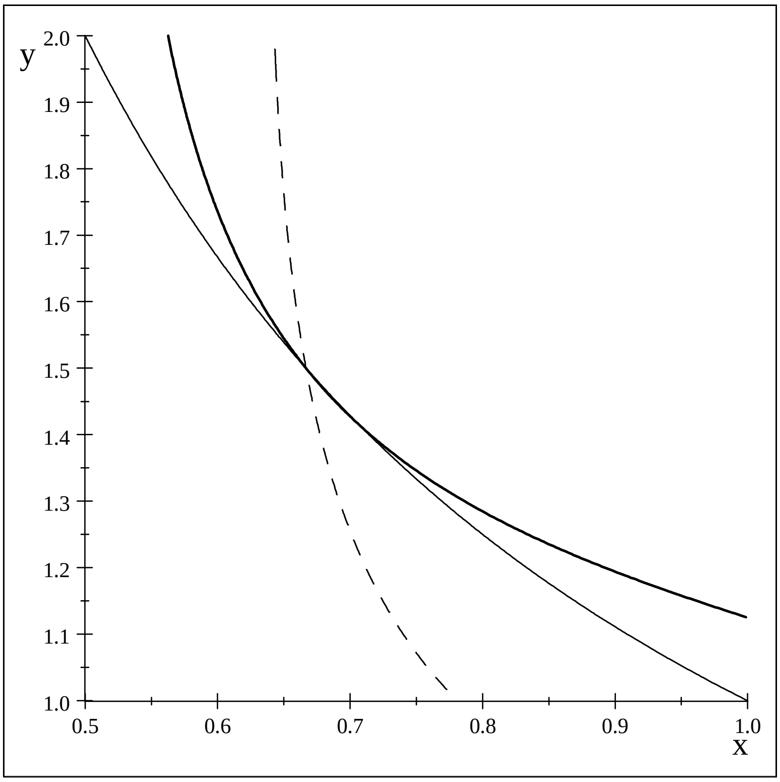
Step 2.2.2 For $\omega \in [1, \frac{1}{\lambda}]$, the inequality $\pi'_H(p_L + \theta) \leq 0$ is satisfied Notice that the equation $2\lambda x^2 - (1 - \lambda\omega + \omega + 2\lambda)\,x + \omega = 0$ has two roots $x_1 = \frac{1}{4\lambda}\,(1 + 2\lambda - \lambda\omega + \omega - X)$ and $x_2 = \frac{1}{4\lambda}\,(1 + 2\lambda - \lambda\omega + \omega + X)$ with $X = \sqrt{(1-\lambda)^2\,\omega^2 - 2\left(3\lambda - 1 + 2\lambda^2\right)\omega + (2\lambda+1)^2}$. Since $x_1 \in (0,1)$ and $x_2 > 1$,[18] we infer that $\pi'_H(p_L + \theta) \leq 0$ if and only if $\hat{x} \geq x_1$.

[18] The inequality $x_1 > 0$ is equivalent to $1 + 2\lambda - \lambda\omega + \omega > X$, which after squaring reduces to $8\lambda\omega > 0$. The inequality $x_1 < 1$ is equivalent to $1 - 2\lambda - \lambda\omega + \omega < X$, which holds if the left hand side is negative, otherwise after squaring we get an equivalent inequality which boils down to $\omega < \frac{1}{\lambda}$. Finally, the inequality $x_2 > 1$ is equivalent to $X > \lambda(2+\omega) - 1 - \omega$, and it is satisfied if the right hand side is negative, otherwise after squaring we get an equivalent inequality which reduces to $\omega < \frac{1}{\lambda}$.

The inequality $\hat{x} \geq x_1$ is equivalent to $\psi(x_1) < 0$ and we find that

$$\begin{aligned}
\psi(x_1) &= \frac{1}{16\lambda^2}\begin{pmatrix} -(1-\lambda)X^3 + (-\lambda - 8\lambda\omega + 3\omega - 2\lambda^2 + 5\lambda^2\omega)X^2 \\ +((7\lambda-3)(\lambda-1)^2\omega^2 - 2\lambda\left(-3\lambda+6\lambda^2+1\right)\omega + 3 - 9\lambda - 4\lambda^3 + 16\lambda^2)X \\ +(3\lambda-1)(\lambda-1)^3\omega^3 + \lambda(1-\lambda)\left(10\lambda^2 - 5\lambda + 3\right)\omega^2 \\ +\left(20\lambda^3 - 47\lambda^2 - 3 + 4\lambda^4 + 18\lambda\right)\omega + (2\lambda+1)\left(4\lambda^3 - 12\lambda^2 + 13\lambda - 2\right) \end{pmatrix} \\
&= \frac{2\lambda-1}{8\lambda^2}\begin{pmatrix} \left(-4\lambda^2\omega + 2\lambda^2\omega^2 - \lambda\omega - 4\lambda\omega^2 + 4\lambda - 1 + \omega + 2\omega^2\right)X \\ -2(1-\lambda)^3\omega^3 + (1-\lambda)(8\lambda^2+7\lambda-3)\omega^2 + 2\lambda(4\lambda^2+5\lambda-5)\omega - 8\lambda^2 - 2\lambda + 1 \end{pmatrix} \\
&= \frac{2\lambda-1}{8\lambda^2}[\alpha_\lambda(\omega) + \beta_\lambda(\omega)X]
\end{aligned}$$

with $\alpha(\lambda,\omega) = 2(1-\lambda)^2\omega^2 - (4\lambda^2 - 1 + \lambda)\omega + 4\lambda - 1$ and $\beta(\lambda,\omega) = -2(1-\lambda)^3\omega^3 + (1-\lambda)(8\lambda^2+7\lambda-3)\omega^2 + 2\lambda(4\lambda^2+5\lambda-5)\omega - 8\lambda^2 - 2\lambda + 1$. We show in the following that $\Psi(x_1) < 0$ in the set $K = \{(\lambda,\omega) : \lambda \in (\frac{1}{2}, 1]$ and $\omega \in [1, \frac{1}{\lambda}]\}$. We use numerical methods to find that $\beta(\lambda,\omega) < 0$ below the medium thick curve, and $\beta(\lambda,\omega) > 0$ above that curve; notice that the set K includes some points (λ,ω) such that $\beta(\lambda,\omega) > 0$, even though it is impossible to see this set in the picture. Furthermore, numerical methods to find that $\alpha(\lambda,\omega) > 0$ to the left of the very thick curve, and $\alpha(\lambda,\omega) < 0$ to the right of that curve. Also notice that the two curves cross at $(\lambda,\omega) = (\frac{2}{3}, \frac{3}{2})$.



To the left of the dashed curve we have $\alpha > 0$ and $\beta < 0$, thus $\alpha X + \beta \leq 0$ is equivalent to $\beta^2 - \alpha^2 X^2 \geq 0$, which reduces to $-14\lambda\omega + 6\omega + 12\lambda - 3 \geq 0$. Let $\mu(\lambda,\omega) \equiv -14\lambda\omega + 6\omega + 12\lambda - 3$ and notice that for each $\lambda > \frac{1}{2}$, μ is decreasing with respect to ω. For $\lambda \leq \frac{2}{3}$, the maximal ω is $\frac{1}{\lambda}$ and $\mu(\lambda, \frac{1}{\lambda}) = -14\lambda\frac{1}{\lambda} + 6\frac{1}{\lambda} + 12\lambda - 3 = \frac{(2-3\lambda)(3-4\lambda)}{\lambda} \geq 0$. For $\lambda \in (\frac{2}{3}, \frac{\sqrt{17}-1}{4}]$, we can solve $\alpha(\lambda,\omega) = 0$ with respect to ω and find that the maximal ω is $\frac{1}{2(2\lambda^2-4\lambda+2)}\left(4\lambda^2+\lambda-1-\sqrt{16\lambda^4-24\lambda^3+65\lambda^2-50\lambda+9}\right)$. It turns out that $\mu(\lambda,\frac{1}{2(2\lambda^2-4\lambda+2)}\left(4\lambda^2+\lambda-1-\sqrt{16\lambda^4-24\lambda^3+65\lambda^2-50\lambda+9}\right))=\frac{(7\lambda-3)\sqrt{16\lambda^4-24\lambda^3+65\lambda^2-50\lambda+9}-9-4\lambda^3-49\lambda^2+46\lambda}{2(1-\lambda)^2}$, which is positive for $\lambda\in(\frac{2}{3},\frac{\sqrt{17}-1}{4}]$.

To the right of the dashed curve, the inequality $\alpha X+\beta\leq 0$ obviously holds if $\alpha<0$ and $\beta<0$. However, there is a very small region in which $\alpha<0$ and $\beta>0$ (it consists of points such that ω is close to $\frac{1}{\lambda}$ and $\frac{2}{3}<\lambda<\frac{1}{\sqrt{2}}$). In such a case, $\alpha X+\beta\leq 0$ is equivalent to $\alpha^2X^2-\beta^2\geq 0$, which reduces to $-\mu(\lambda,\omega)\geq 0$. At the points in the lower part of the region $\beta=0$, hence $-\mu(\lambda,\omega)\geq 0$ holds since it is equivalent to $\alpha^2X^2-\beta^2\geq 0$, that is $\alpha^2X^2\geq 0$. Moreover, the function $-\mu$ is increasing in ω and thus we infer that $-\mu(\lambda,\omega)\geq 0$ is satisfied at each point in the region.

Values of p_{1H} in $[p_L+\theta,p_H+\theta]$. Then $\pi_H'(p_{1H})=(1-\lambda)(\frac{1}{2}+\frac{1}{\theta}(p_H-p_{1H})+\frac{1}{2\theta^2}(p_H-p_{1H})^2)-(1-\lambda)(p_{1H}-c_H)(\frac{1}{\theta}+\frac{1}{\theta^2}(p_H-p_{1H}))$, which is convex in p_{1H} and $\pi_H'(p_L+\theta)=\frac{1-\lambda}{2\lambda-1}[2\lambda x^2-(1-\lambda\omega+\omega+2\lambda)x+\omega]\leq 0$ (proved above) and $\pi_H'(p_H+\theta)=0$. Hence $\pi_H'(p_{1H})<0$ holds for $p_{1H}\in(p_L+\theta,p_H+\theta)$.

Step 2.3 If type L and type H of supplier 2 play p_L and p_H in (3), respectively, then playing p_L is a best reply for type L of supplier 1 The payoff function of type L is $\pi_L(p_{1L})\equiv(p_{1L}-c_L)[\lambda F(p_L-p_{1L})+(1-\lambda)F(p_H-p_{1L})]$, and

$$\pi_L'(p_{1L})=[\lambda f(p_L-p_{1L})+(1-\lambda)f(p_H-p_L)][\frac{\lambda F(p_L-p_{1L})+(1-\lambda)F(p_H-p_{1L})}{\lambda f(p_L-p_{1L})+(1-\lambda)f(p_H-p_{1L})}-p_{1L}+c_L]$$

Step 2.1 reveals that if $\lambda\leq\frac{1}{2}$, then the second factor in $\pi_L'(p_{1L})$ is strictly decreasing in p_{1L} – interpret p_L-p_{1L} as y – for $p_{1L}\in[p_L-\theta,p_H+\theta)$. Therefore $\pi_L'(p_L)=0$ implies $\pi_L'(p_{1L})>0$ for $p_{1L}\in[p_L-\theta,p_L)$ and $\pi_L'(p_{1L})<0$ for $p_{1L}\in[p_L,p_H+\theta)$. If instead $\lambda>\frac{1}{2}$, then the second factor in $\pi_L'(p_{1L})$ is strictly decreasing in p_{1L} for $p_{1L}\in[p_L-\theta,p_H)$; therefore $\pi_L'(p_{1L})>0$ for $p_{1L}\in[p_L-\theta,p_L)$ and $\pi_L'(p_{1L})<0$ for $p_{1L}\in(p_L,p_H]$. For $p_{1L}\in(p_H,p_L+\theta)$ we obtain

$$\begin{aligned}\pi_L'(p_{1L}) &= \frac{1}{2}+\frac{\lambda}{\theta}(p_L-p_{1L}+\frac{1}{2\theta}(p_L-p_{1L})^2)+\frac{1-\lambda}{\theta}(p_H-p_{1L}+\frac{1}{2\theta}(p_H-p_{1L})^2)+\\ &\quad -\frac{p_{1L}-c_L}{\theta}(1+\frac{\lambda}{\theta}(p_L-p_{1L})+\frac{1-\lambda}{\theta}(p_H-p_{1L}))\end{aligned}$$

The inequality $\pi_L'(p_{1L})<0$ holds for any $p_{1L}\in(p_H,p_L+\theta)$ since $c_L<c_H$ implies $\pi_L'(p_{1L})<\pi_H'(p_{1L})$ and we know that $\pi_H'(p_{1L})<0$ for any $p_{1L}\in(p_H,p_L+\theta)$.

Values of p_{1L} in $[p_L+\theta,p_H+\theta]$. Then $\pi_L'(p_{1L})=(1-\lambda)(\frac{1}{2}+\frac{1}{\theta}(p_H-p_{1L})+\frac{1}{2\theta^2}(p_H-p_{1L})^2)-(1-\lambda)(p_{1L}-c_L)(\frac{1}{\theta}+\frac{1}{\theta^2}(p_H-p_{1L}))$ and $\pi_L'(p_{1L})<0$ for any $p_{1L}\in(p_L+\theta,p_H+\theta)$ since $\pi_L'(p_{1L})<\pi_H'(p_{1L})$ (as $c_L<c_H$) and $\pi_H'(p_{1L})<0$ for any $p_{1L}\in(p_L+\theta,p_H+\theta]$.

6.1.2 The case in which $\Delta>\theta$

Step 1 Derivation of equilibrium bids and the payoff of the buyer When $\Delta>\theta$, the first order conditions (13)-(14) boil down to

$$\frac{\lambda}{2}+1-\lambda=(p_L-c_L)\frac{\lambda}{\theta}\qquad\text{and}\qquad\frac{1-\lambda}{2}=(p_H-c_H)\frac{1-\lambda}{\theta}$$

Thus we obtain $p_L=c_L+(\frac{1}{\lambda}-\frac{1}{2})\theta$, $p_H=c_L+\sigma+\frac{1}{2}\theta$ as in (7), and the inequality $\Delta>\theta$ is equivalent to $\sigma>\frac{1}{\lambda}\theta$. Given p_L,p_H, the payoff of type L is $\frac{(2-\lambda)^2}{4\lambda}\theta$ and the payoff of type H is $\frac{1}{4}(1-\lambda)\theta$.

The payoff of the buyer is

$$\begin{aligned}U_C^B &= \lambda^2(E[\max\{q_1,q_2\}]-p_L)+(1-\lambda)^2(E[\max\{q_1,q_2\}]-p_H)+2\lambda(1-\lambda)[E(q_1)-p_L]\\ &= \underline{q}-c_L-\frac{1}{6}(4\lambda^2-16\lambda+11)\theta-(1-\lambda)^2\sigma\end{aligned}$$

Step 2 Proof that p_L, p_H in (6) constitute a BNE if (λ,σ) belongs to $\mathcal{C}$ First notice that $(\lambda,\sigma)\in\mathcal{C}$ implies $p_H-\theta<p_L+\theta$, and therefore for a supplier choosing price z the probability $\eta(z)$ of winning depends on z as follows

$$\begin{aligned}\eta(z) &= \begin{cases}1-\lambda+\lambda F(p_L-z) & \text{for } p_L-\theta\le z<p_L\\ 1-\lambda+\lambda F(p_L-z) & \text{for } p_L\le z<p_H-\theta\\ (1-\lambda)F(p_H-z)+\lambda F(p_L-z) & \text{for } p_H-\theta\le z<p_L+\theta\\ (1-\lambda)F(p_H-z) & \text{for } p_L+\theta\le z<p_H\\ (1-\lambda)F(p_H-z) & \text{for } p_H\le z<p_H+\theta\end{cases}\\ &= \begin{cases}1-\lambda+\lambda(\frac12+\frac1\theta(p_L-z)-\frac{1}{2\theta^2}(p_L-z)^2) & \text{for } p_L-\theta\le z<p_L\\ 1-\lambda+\lambda(\frac12+\frac1\theta(p_L-z)+\frac{1}{2\theta^2}(p_L-z)^2) & \text{for } p_L\le z<p_H-\theta\\ (1-\lambda)(\frac12+\frac1\theta(p_H-z)-\frac{1}{2\theta^2}(p_H-z)^2)+\lambda(\frac12+\frac1\theta(p_L-z)+\frac{1}{2\theta^2}(p_L-z)^2) & \text{for } p_H-\theta\le z<p_L+\theta\\ (1-\lambda)(\frac12+\frac1\theta(p_H-z)-\frac{1}{2\theta^2}(p_H-z)^2) & \text{for } p_L+\theta\le z<p_H\\ (1-\lambda)(\frac12+\frac1\theta(p_H-z)+\frac{1}{2\theta^2}(p_H-z)^2) & \text{for } p_H\le z<p_H+\theta\end{cases}\end{aligned}$$

Step 2.1 If type L and type H of supplier 2 play p_L and p_H in (6), respectively, then playing p_L is a best reply for type L of supplier 1 Consider first type L, for which the payoff from playing z is $\pi_L(z)=(z-c_L)\eta(z)$. We prove that $z=p_L$ is a best reply for type L.

- Values of z in $[p_L-\theta,p_L)=[c_L+(\frac1\lambda-\frac32)\theta, c_L+(\frac1\lambda-\frac12)\theta)$. Then $\pi_L(z)=(z-c_L)(1-\lambda+\lambda(\frac12+\frac1\theta(c_L+(\frac1\lambda-\frac12)\theta-z)-\frac{1}{2\theta^2}(c_L+(\frac1\lambda-\frac12)\theta-z)^2))$ and $\pi_L'(z)=\frac{3\lambda}{2\theta^2}(z-c_L-\frac{2-9\lambda}{6\lambda}\theta)(c_L+(\frac1\lambda-\frac12)\theta-z)$ is positive for each $z\in[c_L+(\frac1\lambda-\frac32)\theta,c_L+(\frac1\lambda-\frac12)\theta)$ since $c_L+(\frac1\lambda-\frac32)\theta-c_L-\frac{2-9\lambda}{6\lambda}\theta>0$.

- Values of z in $[p_L,p_H-\theta)=[c_L+(\frac1\lambda-\frac12)\theta,c_L+\sigma-\frac12\theta)$. Then $\pi_L(z)=(z-c_L)(1-\lambda+\lambda(\frac12+\frac1\theta(c_L+(\frac1\lambda-\frac12)\theta-z)+\frac{1}{2\theta^2}(c_L+(\frac1\lambda-\frac12)\theta-z)^2))$ and $\pi_L'(z)=\frac{3\lambda}{2\theta^2}(c_L+(\frac76+\frac{1}{3\lambda})\theta-z)(c_L+(\frac1\lambda-\frac12)\theta-z)$ is negative for any $z\in(c_L+(\frac1\lambda-\frac12)\theta,c_L+\sigma-\frac12\theta)$ since $\sigma<\frac{5\lambda+1}{3\lambda}\theta$ is equivalent to $c_L+(\frac76+\frac{1}{3\lambda})\theta-(c_L+\sigma-\frac12\theta)>0$.

- Values of z in $[p_H-\theta,p_L+\theta)=[c_L+\sigma-\frac12\theta,c_L+(\frac1\lambda+\frac12)\theta)$. Then $\pi_L(z)=(z-c_L)((1-\lambda)(\frac12+\frac1\theta(c_L+\sigma+\frac12\theta-z)-\frac{1}{2\theta^2}(c_L+\sigma+\frac12\theta-z)^2)+\lambda(\frac12+\frac1\theta(c_L+(\frac1\lambda-\frac12)\theta-z)+\frac{1}{2\theta^2}(c_L+(\frac1\lambda-\frac12)\theta-z)^2))$ with $\pi_L'(z)=\frac{3(2\lambda-1)}{2\theta^2}(z-c_L)^2-\frac{3\theta-2\sigma+2\sigma\lambda}{\theta^2}(z-c_L)+\frac{-4\lambda(1-\lambda)\sigma^2+4\lambda(1-\lambda)\theta\sigma+(4+11\lambda-6\lambda^2)\theta^2}{8\lambda\theta^2}$.

  Case of $\lambda\le\frac12$. Then $\pi_L'(z)\le0$ for any $z\in[c_L+\sigma-\frac12\theta,c_L+(\frac1\lambda+\frac12)\theta)$ since $\pi_L'(c_L+\sigma-\frac12\theta)=\frac{3\lambda}{2\theta^2}(\sigma-\frac\theta\lambda)(\sigma-\frac{5\lambda+1}{3\lambda}\theta)\le0$ and $\pi_L''(z)=\frac{3(2\lambda-1)}{\theta^2}(z-c_L)-\frac{3\theta-2\sigma+2\lambda\sigma}{\theta^2}<0$ for each $z\in[c_L+\sigma-\frac12\theta,c_L+(\frac1\lambda+\frac12)\theta)$, given $\sigma\le\frac{5\lambda+1}{3\lambda}\theta$.

  Case of $\lambda>\frac12$. Then $\pi_L'(c_L+\sigma-\frac12\theta)=\frac{3\lambda}{2\theta^2}(\sigma-\frac\theta\lambda)(\sigma-\frac{5\lambda+1}{3\lambda}\theta)<0$ and π_L' is convex, thus if $\pi_L'(c_L+(\frac1\lambda+\frac12)\theta)\le0$ then $\pi_L'(z)\le0$ for any $z\in[c_L+\sigma-\frac12\theta,c_L+(\frac1\lambda+\frac12)\theta)$. If instead $\pi_L'(c_L+(\frac1\lambda+\frac12)\theta)>0$, then there exists a $\hat z>c_L+(\frac1\lambda+\frac12)\theta$ which is a local maximum point for π_L, and we analyze it in next step.

- Values of z in $[p_L+\theta,p_H)=[c_L+(\frac1\lambda+\frac12)\theta,c_L+\sigma+\frac12\theta)$. Then $\pi_L(z)=(z-c_L)(1-\lambda)(\frac12+\frac1\theta(c_L+\sigma+\frac12\theta-z)-\frac{1}{2\theta^2}(c_L+\sigma+\frac12\theta-z)^2)$ and $\pi_L'(z)=\frac{1-\lambda}{\theta^2}[-\frac32(z-c_L)^2+(2\sigma-\theta)(z-c_L)+\frac18(7\theta^2+4\theta\sigma-4\sigma^2)]$ is decreasing since $\pi_L''(z)=\frac{1-\lambda}{\theta^2}[-3(z-c_L)+2\sigma-\theta]<0$ for any $z\in[c_L+(\frac1\lambda+\frac12)\theta,c_L+\sigma+\frac12\theta)$, given

$\sigma \leq \frac{5\lambda+1}{3\lambda}\theta$. We find that $\pi'_L(c_L + (\frac{1}{\lambda} + \frac{1}{2})\theta) = (1-\lambda)\frac{-\lambda^2\sigma^2+\lambda(4+3\lambda)\theta\sigma-(3+5\lambda)\theta^2}{2\theta^2\lambda^2}$, and if $\pi'_L(c_L + (\frac{1}{\lambda} + \frac{1}{2})\theta) \leq 0$ then $\pi'_L(z) < 0$ for any $z \in [c_L + (\frac{1}{\lambda} + \frac{1}{2})\theta, c_L + \sigma + \frac{1}{2}\theta)$. If instead $\pi'_L(c_L + (\frac{1}{\lambda} + \frac{1}{2})\theta) > 0$, then we find a local maximum for π_L at $\hat{z} \equiv c_L + \frac{2}{3}\sigma - \frac{1}{3}\theta + \frac{1}{6}\sqrt{4\sigma^2 - 4\theta\sigma + 25\theta^2}$, with $\pi_L(\hat{z}) = \frac{1-\lambda}{216\theta^2}[-8\sigma^3 + 12\theta\sigma^2 + 138\sigma\theta^2 - 71\theta^3 + (4\sigma^2 - 4\theta\sigma + 25\theta^2)^{3/2}]$. It turns out that $\pi_L(\hat{z})$ is increasing in σ,[19] and at $\sigma = \frac{5\lambda+1}{3\lambda}\theta$ we obtain $\pi_L(\hat{z}) = \frac{1-\lambda}{5832\lambda^3}(4193\lambda^3 + 1002\lambda^2 - 84\lambda - 8 + (265\lambda^2 + 28\lambda + 4)^{3/2})\theta$, which is smaller than $\frac{(2-\lambda)^2}{4\lambda}\theta$ for any $\lambda > \frac{2}{5}$.

- Values of z in $[p_H, p_H + \theta] = [c_L + \sigma + \frac{1}{2}\theta, c_L + \sigma + \frac{3}{2}\theta]$. Then $\pi_L(z) = (z - c_L)(1-\lambda)(\frac{1}{2} + \frac{1}{\theta}(c_L + \sigma + \frac{1}{2}\theta - z) + \frac{1}{2\theta^2}(c_L + \sigma + \frac{1}{2}\theta - z)^2)$ and $\pi'_L(z) = \frac{3(1-\lambda)}{2\theta^2}(z - c_L)^2 - (1-\lambda)\frac{3\theta+2\sigma}{\theta^2}(z - c_L) + (1-\lambda)\frac{(3\theta+2\sigma)^2}{8\theta^2}$ is negative for any $z \in [c_L + \sigma + \frac{1}{2}\theta, c_L + \sigma + \frac{3}{2}\theta)$ since π'_L is convex and $\pi'_L(c_L + \sigma + \frac{1}{2}\theta) = -\frac{1-\lambda}{\theta}\sigma$ and $\pi'_L(c_L + \sigma + \frac{3}{2}\theta) = 0$.

Step 2.2 If type L and type H of supplier 2 play p_L and p_H in (6), respectively, then playing p_H is a best reply for type H of supplier 1 Now we consider type H, for which the payoff from playing z is $\pi_H(z) = (z - c_H)\eta(z)$. We prove that $z = p_H$ is a best reply for type H. Since the cost of type H is $c_H = c_L + \sigma$, we do not need to consider $z < c_L + \sigma$. Precisely, we start with $z \in [c_L + \sigma, c_L + (\frac{1}{\lambda} + \frac{1}{2})\theta)$ if $\sigma < (\frac{1}{\lambda} + \frac{1}{2})\theta$; otherwise we start with $z \in [c_L + \sigma, c_L + \sigma + \frac{1}{2}\theta)$.

- Values of z in $[c_L + \sigma, p_L + \theta) = [c_L + \sigma, c_L + (\frac{1}{\lambda} + \frac{1}{2})\theta)$. Then $\pi_H(z) = (z - c_L - \sigma)((1-\lambda)(\frac{1}{2} + \frac{1}{\theta}(c_L + \sigma + \frac{1}{2}\theta - z) - \frac{1}{2\theta^2}(c_L + \sigma + \frac{1}{2}\theta - z)^2) + \lambda(\frac{1}{2} + \frac{1}{\theta}(c_L + (\frac{1}{\lambda} - \frac{1}{2})\theta - z) + \frac{1}{2\theta^2}(c_L + (\frac{1}{\lambda} - \frac{1}{2})\theta - z)^2))$ and $\pi'_H(z) = \frac{3(2\lambda-1)}{2\theta^2}(z - \sigma - c_L)^2 - \frac{3\theta-2\sigma\lambda}{\theta^2}(z - \sigma - c_L) + \frac{[(\lambda+2)\theta-2\lambda\sigma]^2+7\lambda(1-\lambda)\theta^2}{8\lambda\theta^2}$.
  Case of $\lambda \leq \frac{1}{2}$. Then $\pi'_H(z) > 0$ for any $z \in [c_L + \sigma, c_L + (\frac{1}{\lambda} + \frac{1}{2})\theta)$ since π'_H is concave and $\pi'_H(c_L + \sigma) = \frac{[(\lambda+2)\theta-2\lambda\sigma]^2+7\lambda(1-\lambda)\theta^2}{8\lambda\theta^2} > 0$, $\pi'_H(c_L + (\frac{1}{\lambda} + \frac{1}{2})\theta) = (1-\lambda)(\sigma\lambda - \theta)\frac{(5\lambda+3)\theta-3\lambda\sigma}{2\theta^2\lambda^2} \geq 0$.
  Case of $\lambda > \frac{1}{2}$. We have seen above that $\pi'_H(c_L + (\frac{1}{\lambda} + \frac{1}{2})\theta) \geq 0$, and $\pi''_H(z) = \frac{3(2\lambda-1)}{\theta^2}(z - \sigma - c_L) - \frac{3\theta-2\sigma\lambda}{\theta^2}$. We can prove that if $\lambda \in (\frac{1}{2}, \frac{5}{6}]$, then $\pi''_H(z) \leq 0$ for any $z \in [c_L + \sigma, c_L + (\frac{1}{\lambda} + \frac{1}{2})\theta)$, which implies that $\pi'_H(z) > 0$ for any $z \in [c_L + \sigma, c_L + (\frac{1}{\lambda} + \frac{1}{2})\theta)$. Precisely, $\pi''_H(c_L + (\frac{1}{\lambda} + \frac{1}{2})\theta) = \frac{1}{2\lambda\theta^2}(2\lambda(3-4\lambda)\sigma + 3(2\lambda^2 + \lambda - 2)\theta)$ and (i) if $\frac{1}{2} < \lambda < \frac{3}{4}$, then $2\lambda(3-4\lambda)\sigma + 3(2\lambda^2 + \lambda - 2)\theta \leq 2\lambda(3-4\lambda)\frac{5\lambda+1}{3\lambda}\theta + 3(2\lambda^2 + \lambda - 2)\theta = -\frac{1}{3}(22\lambda^2 - 31\lambda + 12)\theta < 0$; (ii) if $\frac{3}{4} \leq \lambda < \frac{\sqrt{17}-1}{4}$, then $2\lambda(3-4\lambda)\sigma + 3(2\lambda^2 + \lambda - 2)\theta < 0$ since $3 - 4\lambda < 0$ and $2\lambda^2 + \lambda - 2 < 0$; (iii) if $\frac{\sqrt{17}-1}{4} \leq \lambda \leq \frac{5}{6}$, then $2\lambda(3-4\lambda)\sigma + 3(2\lambda^2 + \lambda - 2)\theta \leq 2\lambda(3-4\lambda)\frac{\theta}{\lambda} + 3(2\lambda^2 + \lambda - 2)\theta = -\lambda(5 - 6\lambda)\theta \leq 0$. If $\lambda > \frac{5}{6}$, we need to study the sign of $\pi''_H(c_L + (\frac{1}{\lambda} + \frac{1}{2})\theta)$. This is positive for $\lambda > \frac{5}{6}$, at $\sigma = \frac{\theta}{\lambda}$; for a larger σ, it may become negative. Hence π'_H has a minimum in the interval $[c_L + \sigma, c_L + (\frac{1}{\lambda} + \frac{1}{2})\theta)$ if $\lambda > \frac{5}{6}$ and σ is close to $\frac{\theta}{\lambda}$. Precisely, it is necessary that $\sigma < \frac{3}{2}\frac{2\lambda^2+\lambda-2}{\lambda(4\lambda-3)}\theta$, and we find that the minimum value is $\frac{-4\lambda^2(3-2\lambda)\sigma^2+12(2+\lambda-2\lambda^2)\theta\lambda\sigma-3\theta^2(4-28\lambda^2+15\lambda+12\lambda^3)}{24\lambda(2\lambda-1)\theta^2}$, which is positive for σ between $(\frac{9}{5}\lambda - \frac{3}{10})\theta$ and $\frac{3}{2}\frac{2\lambda^2+\lambda-2}{\lambda(4\lambda-3)}\theta$, since $(8\lambda^3 - 12\lambda^2)((\frac{9}{5}\lambda - \frac{3}{10})\theta)^2 + (12\theta\lambda^2 - 24\theta\lambda^3 + 24\theta\lambda)((\frac{9}{5}\lambda - \frac{3}{10})\theta) + 84\theta^2\lambda^2 - 45\lambda\theta^2 - 36\theta^2\lambda^3 - 12\theta^2 = \frac{3}{25}(1-\lambda)(6\lambda+1)(6\lambda-5)(20 + 11\lambda - 6\lambda^2)\theta^2 > 0$.

- Values of z in $[p_L + \theta, p_H) = [c_L + (\frac{1}{\lambda} + \frac{1}{2})\theta, c_L + \sigma + \frac{1}{2}\theta)$. Then $\pi_H(z) = (z - c_L - \sigma)(1-\lambda)(\frac{1}{2} + \frac{1}{\theta}(c_L + \sigma + \frac{1}{2}\theta - z) - \frac{1}{2\theta^2}(c_L + \sigma + \frac{1}{2}\theta - z)^2)$, and $\pi'_H(z) = \frac{3}{2}\frac{1-\lambda}{\theta^2}(z - c_L - \sigma + \frac{7}{6}\theta)(c_L + \sigma + \frac{1}{2}\theta - z)$ is positive for any $z \in [c_L + (\frac{1}{\lambda} + \frac{1}{2}), c_L + \sigma + \frac{1}{2}\theta)$ since $c_L + (\frac{1}{\lambda} + \frac{1}{2})\theta - c_L - \sigma + \frac{7}{6}\theta > 0$ given $\sigma < \frac{5\lambda+1}{3\lambda}\theta$.[20]

[19] First set $\omega = \frac{\sigma}{\theta}$ and notice that $\omega \leq \frac{5\lambda+1}{3\lambda} \leq \frac{5}{2}$ for any $(\lambda, \sigma) \in \mathcal{C}$. Then notice that $\frac{1}{\theta}\pi_L(\hat{z}) = -8\omega^3 + 12\omega^2 + 138\omega - 71 + (4\omega^2 - 4\omega + 25)^{3/2}$ which is increasing in ω for $\omega \in [1, \frac{5}{2}]$.

[20] In case that $\sigma > (\frac{1}{\lambda} + \frac{1}{2})\theta$, our proof for values of z in $[c_L + (\frac{1}{\lambda} + \frac{1}{2})\theta, c_L + \sigma + \frac{1}{2}\theta)$ covers the set of values of z in $[c_L + \sigma, c_L + \sigma + \frac{1}{2}\theta)$.

- Values of z in $[p_H, p_H+\theta] = [c_L+\sigma+\frac{1}{2}\theta, c_L+\sigma+\frac{3}{2}\theta]$. Then $\pi_H(z) = (z-c_L-\sigma)(1-\lambda)(\frac{1}{2}+\frac{1}{\theta}(c_L+\sigma+\frac{1}{2}\theta-z)+\frac{1}{2\theta^2}(c_L+\sigma+\frac{1}{2}\theta-z)^2)$ and $\pi'_H(z) = \frac{3(1-\lambda)}{2\theta^2}(z-c_L-\sigma-\frac{3}{2}\theta)(z-c_L-\sigma-\frac{1}{2}\theta) \leq 0$ for any $z \in (c_L+\sigma+\frac{1}{2}\theta, c_L+\sigma+\frac{3}{2}\theta]$.

6.1.3 Proof of Proposition 1(iii)

When $\lambda = \frac{1}{2}$, a pure strategy BNE exists for any $\omega \leq 2\sqrt{3}-1$. For values of ω larger than $2\sqrt{3}-1$, a mixed-strategy symmetric BNE exists since for each type i_k of supplier the payoff function in (2) is continuous in bids. Thus we can apply a standard existence result which can be found, for instance, in Fudenberg and Tirole, 1991 (Theorem 1.3 in Subsection 1.3.3).

The strategy profiles described by Proposition 1(ii) are such that each type L bids p_L^a with probability μ, bids p_L^b with probability $1-\mu$, and each type H bids p_H. We need to prove that for each $\omega \in \{\frac{5}{2}, 3, \frac{7}{2}\}$, the corresponding strategy profile in Proposition 1(ii) is a BNE. First notice that in each of the three cases we have

$$p_L^a - \theta < p_L^b - \theta < p_L^a < p_H - \theta < p_L^b < p_L^a + \theta < p_H < p_L^b + \theta < p_H + \theta \tag{19}$$

Therefore the payoff of type L of supplier 1 from bidding $p_{1L} \in [p_L^a - \theta, p_H + \theta]$, denoted with $\pi_{1L}(p_{1L})$, is given by

$$\begin{cases}
(p_{1L}-c_L)\{\frac{1}{2}+\frac{\mu}{2}[\frac{1}{2}+\frac{p_L^a-p_{1L}}{\theta}-\frac{(p_L^a-p_{1L})^2}{2\theta^2}]+\frac{1-\mu}{2}\} & \text{if } p_{1L} \in [p_L^a-\theta, p_L^b-\theta) \\
(p_{1L}-c_L)\{\frac{1}{2}+\frac{\mu}{2}[\frac{1}{2}+\frac{p_L^a-p_{1L}}{\theta}-\frac{(p_L^a-p_{1L})^2}{2\theta^2}]+\frac{1-\mu}{2}[\frac{1}{2}+\frac{p_L^b-p_{1L}}{\theta}-\frac{(p_L^b-p_{1L})^2}{2\theta^2}]\} & \text{if } p_{1L} \in [p_L^b-\theta, p_L^a) \\
(p_{1L}-c_L)\{\frac{1}{2}+\frac{\mu}{2}\frac{(\theta+p_L^a-p_{1L})^2}{2\theta^2}+\frac{1-\mu}{2}[\frac{1}{2}+\frac{p_L^b-p_{1L}}{\theta}-\frac{(p_L^b-p_{1L})^2}{2\theta^2}]\} & \text{if } p_{1L} \in [p_L^a, p_H-\theta) \\
(p_{1L}-c_L)\{\frac{1}{2}[\frac{1}{2}+\frac{p_H-p_{1L}}{\theta}-\frac{(p_H-p_{1L})^2}{2\theta^2}]+\frac{\mu}{2}\frac{(\theta+p_L^a-p_{1L})^2}{2\theta^2}+\frac{1-\mu}{2}[\frac{1}{2}+\frac{p_L^b-p_{1L}}{\theta}-\frac{(p_L^b-p_{1L})^2}{2\theta^2}]\} & \text{if } p_{1L} \in [p_H-\theta, p_L^b) \\
(p_{1L}-c_L)\{\frac{1}{2}[\frac{1}{2}+\frac{p_H-p_{1L}}{\theta}-\frac{(p_H-p_{1L})^2}{2\theta^2}]+\frac{\mu}{2}\frac{(\theta+p_L^a-p_{1L})^2}{2\theta^2}+\frac{1-\mu}{2}\frac{(\theta+p_L^b-p_{1L})^2}{2\theta^2}\} & \text{if } p_{1L} \in [p_L^b, p_L^a+\theta) \\
(p_{1L}-c_L)\{\frac{1}{2}[\frac{1}{2}+\frac{p_H-p_{1L}}{\theta}-\frac{(p_H-p_{1L})^2}{2\theta^2}]+\frac{1-\mu}{2}\frac{(\theta+p_L^b-p_{1L})^2}{2\theta^2}\} & \text{if } p_{1L} \in [p_L^a+\theta, p_H) \\
(p_{1L}-c_L)[\frac{1}{2}\frac{(\theta+p_H-p_{1L})^2}{2\theta^2}+\frac{1-\mu}{2}\frac{(\theta+p_L^b-p_{1L})^2}{2\theta^2}] & \text{if } p_{1L} \in [p_H, p_L^b+\theta) \\
(p_{1L}-c_L)\frac{1}{2}\frac{(\theta+p_H-p_{1L})^2}{2\theta^2} & \text{if } p_{1L} \in [p_L^b+\theta, p_H+\theta]
\end{cases}$$

Likewise, the payoff of type H of supplier 1 from bidding $p_{1H} \in [p_L^a-\theta, p_H+\theta]$, denoted with $\pi_{1H}(p_{1H})$, is given by

$$\begin{cases}
(p_{1H}-c_H)\{\frac{1}{2}+\frac{\mu}{2}[\frac{1}{2}+\frac{p_L^a-p_{1H}}{\theta}-\frac{(p_L^a-p_{1H})^2}{2\theta^2}]+\frac{1-\mu}{2}\} & \text{if } p_{1H} \in [p_L^a-\theta, p_L^b-\theta) \\
(p_{1H}-c_H)\{\frac{1}{2}+\frac{\mu}{2}[\frac{1}{2}+\frac{p_L^a-p_{1H}}{\theta}-\frac{(p_L^a-p_{1H})^2}{2\theta^2}]+\frac{1-\mu}{2}[\frac{1}{2}+\frac{p_L^b-p_{1H}}{\theta}-\frac{(p_L^b-p_{1H})^2}{2\theta^2}]\} & \text{if } p_{1H} \in [p_L^b-\theta, p_L^a) \\
(p_{1H}-c_H)\{\frac{1}{2}+\frac{\mu}{2}\frac{(\theta+p_L^a-p_{1H})^2}{2\theta^2}+\frac{1-\mu}{2}[\frac{1}{2}+\frac{p_L^b-p_{1H}}{\theta}-\frac{(p_L^b-p_{1H})^2}{2\theta^2}]\} & \text{if } p_{1H} \in [p_L^a, p_H-\theta) \\
(p_{1H}-c_H)\{\frac{1}{2}[\frac{1}{2}+\frac{p_H-p_{1H}}{\theta}-\frac{(p_H-p_{1H})^2}{2\theta^2}]+\frac{\mu}{2}\frac{(\theta+p_L^a-p_{1H})^2}{2\theta^2}+\frac{1-\mu}{2}[\frac{1}{2}+\frac{p_L^b-p_{1H}}{\theta}-\frac{(p_L^b-p_{1H})^2}{2\theta^2}]\} & \text{if } p_{1H} \in [p_H-\theta, p_L^b) \\
(p_{1H}-c_H)\{\frac{1}{2}[\frac{1}{2}+\frac{p_H-p_{1H}}{\theta}-\frac{(p_H-p_{1H})^2}{2\theta^2}]+\frac{\mu}{2}\frac{(\theta+p_L^a-p_{1H})^2}{2\theta^2}+\frac{1-\mu}{2}\frac{(\theta+p_L^b-p_{1H})^2}{2\theta^2}\} & \text{if } p_{1H} \in [p_L^b, p_L^a+\theta) \\
(p_{1H}-c_H)\{\frac{1}{2}[\frac{1}{2}+\frac{p_H-p_{1H}}{\theta}-\frac{(p_H-p_{1H})^2}{2\theta^2}]+\frac{1-\mu}{2}\frac{(\theta+p_L^b-p_{1H})^2}{2\theta^2}\} & \text{if } p_{1H} \in [p_L^a+\theta, p_H) \\
(p_{1H}-c_H)[\frac{1}{2}\frac{(\theta+p_H-p_{1H})^2}{2\theta^2}+\frac{1-\mu}{2}\frac{(\theta+p_L^b-p_{1H})^2}{2\theta^2}] & \text{if } p_{1H} \in [p_H, p_L^b+\theta) \\
(p_{1H}-c_H)\frac{1}{2}\frac{(\theta+p_H-p_{1H})^2}{2\theta^2} & \text{if } p_{1H} \in [p_L^b+\theta, p_H+\theta)
\end{cases}$$

We need to prove that for each $\omega \in \{\frac{5}{2}, 3, \frac{7}{2}\}$, the prices p_L^a, p_L^b given by Proposition 1(iii) are both maximum points for π_{1L}, and that p_H is a maximum point for π_{1H}.[21]

We start by considering type L of supplier 1, and in order to simplify notation we define y as $\frac{p_{1L}-c_L}{\theta}$, and write the profit of type L as a function of y: $\tilde{\pi}_{1L}(y) \equiv \frac{1}{\theta}\pi_{1L}(c_L + y\theta)$ (we have inserted the factor $\frac{1}{\theta}$ – which is irrelevant from the point of view of the incentives of type L – in order to get a simpler expression for $\tilde{\pi}_{1L}$ and $\tilde{\pi}'_{1L}$). Furthermore, we define $a \equiv \frac{p_L^a - c_L}{\theta}$, $b \equiv \frac{p_L^b - c_L}{\theta}$, and $h \equiv \frac{p_H - c_H}{\theta}$. Then $\tilde{\pi}_{1L}$ is defined as follows as a function of $y \in [a-1, \omega+h+1]$

$$\tilde{\pi}_{1L}(y) = \begin{cases} y[\frac{1}{2} + \frac{\mu}{2}(\frac{1}{2} + a - y - \frac{(a-y)^2}{2}) + \frac{1-\mu}{2}] & \text{if } y \in [a-1, b-1) \\ y\{\frac{1}{2} + \frac{\mu}{2}[\frac{1}{2} + a - y - \frac{(a-y)^2}{2}] + \frac{1-\mu}{2}[\frac{1}{2} + b - y - \frac{(b-y)^2}{2}]\} & \text{if } y \in [b-1, a) \\ y\{\frac{1}{2} + \frac{\mu}{2}\frac{(a+1-y)^2}{2} + \frac{1-\mu}{2}[\frac{1}{2} + b - y - \frac{(b-y)^2}{2}]\} & \text{if } y \in [a, h+\omega-1] \\ y\{\frac{1}{2}[\frac{1}{2} + \omega + h - y - \frac{(\omega+h-y)^2}{2}] + \frac{\mu}{2}\frac{(a+1-y)^2}{2} + \frac{1-\mu}{2}[\frac{1}{2} + b - y - \frac{(b-y)^2}{2}]\} & \text{if } y \in [\omega+h-1, b) \\ y\{\frac{1}{2}[\frac{1}{2} + \omega + h - y - \frac{(\omega+h-y)^2}{2}] + \frac{\mu}{2}\frac{(a+1-y)^2}{2} + \frac{1-\mu}{2}\frac{(b+1-y)^2}{2}\} & \text{if } y \in [b, a+1) \\ y\{\frac{1}{2}[\frac{1}{2} + \omega + h - y - \frac{(\omega+h-y)^2}{2}] + \frac{1-\mu}{2}\frac{(b+1-y)^2}{2}\} & \text{if } y \in [a+1, \omega+h) \\ y[\frac{1}{2}\frac{(1+\omega+h-y)^2}{2} + \frac{1-\mu}{2}\frac{(b+1-y)^2}{2}] & \text{if } y \in [\omega+h, b+1) \\ y\frac{1}{2}\frac{(1+\omega+h-y)^2}{2} & \text{if } y \in [b+1, \omega+h+1] \end{cases}$$

and therefore

$$\tilde{\pi}'_{1L}(y) = \begin{cases} -\frac{3}{4}\mu y^2 - \mu(1-a)y + 1 - \frac{1}{4}\mu(1-a)^2 & \text{if } y \in [a-1, b-1) \\ -\frac{3}{4}y^2 + (\mu a + b - 1 - \mu b)y + \frac{3}{4} + \frac{\mu}{4}a(2-a) + \frac{1-\mu}{4}b(2-b) & \text{if } y \in [b-1, a) \\ (\frac{3}{2}\mu - \frac{3}{4})y^2 + (b - \mu a - 1 - \mu b)y + \frac{3}{4} + \frac{\mu}{4}a(2+a) + \frac{1-\mu}{4}b(2-b) & \text{if } y \in [a, \omega+h-1) \\ \begin{pmatrix} \frac{3}{2}(\mu-1)y^2 + (\omega + h - 2 - \mu a + b - \mu b)y \\ +\frac{1}{2}(1+\omega+h) - \frac{1}{4}(\omega+h)^2 + \frac{\mu}{4}a(2+a) + \frac{1-\mu}{4}b(2-b) \end{pmatrix} & \text{if } y \in [\omega+h-1, b) \\ (\omega + h - 2 - \mu a - b + \mu b)y + \frac{1}{2}(1+\omega+h) - \frac{1}{4}(\omega+h)^2 + \frac{\mu}{4}a(2+a) + \frac{1-\mu}{4}b(2+b) & \text{if } y \in [b, a+1) \\ -\frac{3}{4}\mu y^2 + (\omega + h - 2 - b + \mu + \mu b)y + \frac{1}{4} + \frac{1}{2}\omega + \frac{1}{2}h - \frac{1}{4}(\omega+h)^2 + \frac{1}{4}(1-\mu)(b+1)^2 & \text{if } y \in [a+1, \omega+h) \\ (\frac{3}{2} - \frac{3}{4}\mu)y^2 + (\mu b - \omega - h - 2 - b + \mu)y + \frac{1}{4}(1+\omega+h)^2 + \frac{1}{4}(1-\mu)(b+1)^2 & \text{if } y \in [\omega+h, b+1) \\ \frac{3}{4}(y - \frac{1+h+\omega}{3})(y - 1 - h - \omega) & \text{if } y \in [b+1, \omega+h+1] \end{cases}$$

The property that p_L^a, p_L^b are both maximum points for π_{1L} is equivalent to the property that a, b are both maximum points for $\tilde{\pi}_{1L}$, and in order to prove this property we examine the sign of $\tilde{\pi}'_{1L}$. For instance, in

[21] Therefore the following conditions are necessary (but not sufficient): $\pi'_{1L}(p_L^a) = \pi'_{1L}(p_L^b) = 0$, $\pi_{1L}(p_L^a) = \pi_{1L}(p_L^b)$, $\pi'_{1H}(p_H) = 0$, and indeed in order to find p_L^a, p_L^b, μ, p_H given by Proposition 1(iii) we have solved (using numerical methods) the system given by these four equations, for $\omega \in \{\frac{5}{2}, 3, \frac{7}{2}\}$.

the case of $\omega = \frac{5}{2}$ we have $a = 1.515976$, $b = 2.087441$, $\mu = 0.987465$, $h = 0.499498$. Then

$$\tilde{\pi}'_{1L}(y) = \begin{cases} -0.74059875y^2 + 0.5095082408y + 0.934276494 & \text{if } y \in [0.515976, 1.087441) \\ -0.75y^2 + 0.523139314y + 0.9305707471 & \text{if } y \in [1.087441, 1.515976) \\ 0.7311975y^2 - 2.470807168y + 2.0652585 & \text{if } y \in [1.515976, 1.999498) \\ -0.0188025y^2 - 0.471309168y + 1.065760437 & \text{if } y \in [1.999498, 2.087441) \\ -0.523641314y + 1.093070504 & \text{if } y \in [2.087441, 2.515976) \\ -0.74059875y^2 + 1.960796927y - 0.4696262432 & \text{if } y \in [2.515976, 2.999498) \\ 0.75940125y^2 - 4.038199073y + 4.028867883 & \text{if } y \in [2.999498, 3.087441) \\ \frac{3}{4}y^2 - 3.999498y + 3.998996063 & \text{if } y \in [3.087441, 3.999498] \end{cases}$$

and it is simple to see that $\tilde{\pi}'_{1L}(y) > 0$ in $[0.515976, 1.515976)$, $\tilde{\pi}'_{1L}(y) < 0$ in $(1.515976, 1.86315)$, $\tilde{\pi}'_{1L}(y) > 0$ in $(1.86315, 2.087441)$, $\tilde{\pi}'_{1L}(y) < 0$ in $(2.087441, 3.999498)$, and $\tilde{\pi}_{1L}(1.515976) = \tilde{\pi}_{1L}(2.087441) = 1.14086$. A similar argument applies when $\omega \in \{3, \frac{7}{2}\}$.

Regarding type H, we define y as $\frac{p_{1H} - c_H}{\theta}$ and write the profit of type H as a function of y: $\tilde{\pi}_{1H}(y) \equiv \frac{1}{\theta}\pi_{1H}(c_H + y\theta)$; again, the factor $\frac{1}{\theta}$ is irrelevant from the point of view of the incentives of type H. Then $\tilde{\pi}_{1H}$ is defined as follows as a function of $y \in [a - 1 - \omega, h + 1]$:

$$\tilde{\pi}_{1H}(y) = \begin{cases} y\{\frac{1}{2} + \frac{\mu}{2}[\frac{1}{2} + a - y - \omega - \frac{(a-\omega-y)^2}{2}] + \frac{1-\mu}{2}\} & \text{if } y \in [a-1-\omega, b-1-\omega) \\ y\{\frac{1}{2} + \frac{\mu}{2}[\frac{1}{2} + a - y - \omega - \frac{(a-\omega-y)^2}{2}] + \frac{1-\mu}{2}[\frac{1}{2} + b - y - \omega - \frac{(b-y-\omega)^2}{2}]\} & \text{if } y \in [b-1-\omega, a-\omega) \\ y\{\frac{1}{2} + \frac{\mu}{2}\frac{(1+a-\omega-y)^2}{2} + \frac{1-\mu}{2}[\frac{1}{2} + b - y - \omega - \frac{(b-y-\omega)^2}{2}]\} & \text{if } y \in [a-\omega, h-1) \\ y\{\frac{1}{2}[\frac{1}{2} + h - y - \frac{(h-y)^2}{2}] + \frac{\mu}{2}\frac{(1+a-\omega-y)^2}{2} + \frac{1-\mu}{2}[\frac{1}{2} + b - y - \omega - \frac{(b-y-\omega)^2}{2}]\} & \text{if } y \in [h-1, b-\omega) \\ y\{\frac{1}{2}[\frac{1}{2} + h - y - \frac{(h-y)^2}{2}] + \frac{\mu}{2}\frac{(1+a-\omega-y)^2}{2} + \frac{1-\mu}{2}\frac{(1+b-y-\omega)^2}{2}\} & \text{if } y \in [b-\omega, a-\omega+1) \\ y\{\frac{1}{2}[\frac{1}{2} + h - y - \frac{(h-y)^2}{2}] + \frac{1-\mu}{2}\frac{(1+b-y-\omega)^2}{2}\} & \text{if } y \in [a-\omega+1, h) \\ y[\frac{1}{2}\frac{(h+1-y)^2}{2} + \frac{1-\mu}{2}\frac{(1+b-y-\omega)^2}{2}] & \text{if } y \in [h, b-\omega+1) \\ y\frac{1}{2}\frac{(h+1-y)^2}{2} & \text{if } y \in [b-\omega+1, h+1] \end{cases}$$

and therefore

$$\tilde{\pi}'_{1H}(y) = \begin{cases} -\frac{3}{4}\mu y^2 + \mu\left(a-\omega-1\right)y + 1 - \frac{1}{4}\mu(a-\omega-1)^2 & \text{if } y \in (a-1-\omega, b-1-\omega) \\ \left(\begin{array}{c} -\frac{3}{4}y^2 + \left(\mu a - 1 + b - \omega - \mu b\right)y \\ +\frac{3}{4} + \frac{1}{4}\mu(a-\omega)(2-a+\omega) + \frac{1}{4}(1-\mu)(b-\omega)(2-b+\omega) \end{array}\right) & \text{if } y \in [b-1-\omega, a-\omega) \\ \left(\begin{array}{c} (\frac{3}{2}\mu - \frac{3}{4})y^2 + \left(b - 1 - \omega - \mu a - \mu b + 2\mu\omega\right)y \\ +\frac{3}{4} + \frac{1}{4}\mu(a-\omega)(2+a-\omega) + \frac{1}{4}\left(1-\mu\right)\left(b-\omega\right)(2-b+\omega) \end{array}\right) & \text{if } y \in [a-\omega, h-1] \\ \left(\begin{array}{c} \frac{3}{2}(\mu-1)y^2 + \left(h - 2 - \mu a + 2\mu\omega + b - \omega - \mu b\right)y \\ +\frac{2+2h-h^2}{4} + \frac{1}{4}\mu(a-\omega)(2+a-\omega) + \frac{1}{4}(1-\mu)(b-\omega)(2-b+\omega) \end{array}\right) & \text{if } y \in [h-1, b-\omega) \\ \left(\begin{array}{c} (\omega + h - 2 - \mu a - b + \mu b)y + \frac{1}{4}(2+2h-h^2) \\ +\frac{1}{4}\mu(a-\omega)(2+a-\omega) + \frac{1}{4}(1-\mu)(b-\omega)(2+b-\omega) \end{array}\right) & \text{if } y \in [b-\omega, a-\omega+1) \\ \left(\begin{array}{l} -\frac{3}{4}\mu y^2 + (h - 2 + \mu + (\mu-1)(b-\omega))y \\ +\frac{1}{4}(1+2h-h^2) + \frac{1}{4}\left(1-\mu\right)\left(b+1-\omega\right)^2 \end{array}\right) & \text{if } y \in [a-\omega+1, h) \\ \left(\begin{array}{c} \frac{3}{4}(2-\mu)y^2 + (-h - 2 + \mu + (\mu-1)(b-\omega))y \\ +\frac{1}{4}(h+1)^2 + \frac{1}{4}(1-\mu)(b+1-\omega)^2 \end{array}\right) & \text{if } y \in [h, b-\omega+1) \\ \frac{3}{4}\left(\frac{1+h}{3} - y\right)\left(1+h-y\right) & \text{if } y \in [b-\omega+1, h+1] \end{cases}$$

The property that p_H is a maximum point for π_{1H} is equivalent to the property that h is a maximum point for $\tilde{\pi}_{1H}$, and in order to prove this property we examine the sign of $\tilde{\pi}'_{1H}$. For instance, in the case of $\omega = \frac{5}{2}$ we have $a = 1.515976$, $b = 2.087441$, $\mu = 0.987465$, $h = 0.499498$. Clearly, a type H will not choose $y \leq 0$ and $\tilde{\pi}'_{1H}(y)$ is defined as follows for $y > 0$:

$$\tilde{\pi}'_{1H}(y) = \begin{cases} -0.523641314y + 0.4385188615 & \text{if } y \in [0, 0.015976) \\ -0.74059875y^2 - .5078655729y + 0.4384558532 & \text{if } y \in [0.015976, 0.499498) \\ 0.75940125y^2 - 1.506861573y + 0.5632049792 & \text{if } y \in [0.499498, 0.587441) \\ \frac{3}{4}y^2 - 1.499498y + 0.5621235629 & \text{if } y \in [0.587441, 1.499498] \end{cases}$$

and it is simple to see that $\tilde{\pi}'_{1H}(y) > 0$ in $[0, 0.499498)$, $\tilde{\pi}'_{1H}(y) < 0$ in $(0.499498, 1.499498]$. A similar argument applies when $\omega \in \{3, \frac{7}{2}\}$.

In order to evaluate U_C^B in the BNE described by Proposition 1(ii), for $\omega \in \{\frac{5}{2}, 3, \frac{7}{2}\}$, notice that

$$\begin{aligned} U_C^B &= \frac{1}{4}\left[\mu^2 E(\max\{q_1, q_2\} - p_L^a) + (1-\mu)^2 E(\max\{q_1, q_2\} - p_L^b) + 2\mu(1-\mu)E(\max\{q_1 - p_L^a, q_2 - p_L^b\})\right] \\ &\quad \frac{1}{4}E(\max\{q_1, q_2\} - p_H) + \frac{1}{2}\left[\mu E(q_1 - p_L^a) + (1-\mu)E(\max\{q_1 - p_L^b, q_2 - p_H\})\right] \end{aligned}$$

Moreover, $E(\max\{q_1, q_2\}) = \underline{q} + \frac{2}{3}\theta$ and, given x and y which satisfy $x < y < \theta + x$, we find that $E(\max\{q_1 - x, q_2 - y\}) = \int_{\underline{q}}^{\underline{q}+\theta+x-y} \int_{q_1+y-x}^{\underline{q}+\theta} (q_2 - y)\frac{1}{\theta^2} dq_2 dq_1 + \int_{\underline{q}}^{\underline{q}+\theta+x-y} \int_{\underline{q}}^{q_1+y-x} (q_1 - x)\frac{1}{\theta^2} dq_2 dq_1 + \int_{\underline{q}+\theta+x-y}^{\underline{q}+\theta} \int_{\underline{q}}^{\underline{q}+\theta} (q_1 - x)\frac{1}{\theta^2} dq_2 dq_1$. Then for each $\omega \in \{\frac{5}{2}, 3, \frac{7}{2}\}$ we obtain U_C^B as described by Proposition 1(iii).

6.2 Proof of Proposition 2

In this proof we use SA to denote the standard auction environment described in subsection 3.2. In that setting there exists a symmetric BNE in which both bidders bid according to a function β if and only if

$$\text{for any } v_i \qquad \begin{array}{c} (v_i - b_i)\Pr\{b_i > \beta(v_j) \text{ for } j \neq i\} \\ \text{is maximized with respect to } b_i \text{ at } b_i = \beta(v_i) \end{array} \tag{20}$$

Step 1 For any symmetric BNE in the PR setting there exists a symmetric BNE in the SA environment, and viceversa.

Proof First assume that P satisfies (8) and notice from (9)-(10) that the optimal score offered by type c_i, q_i, $q_i - P(c_i, q_i)$, depends only on v_i. We denote by $S(v_i)$ this score, which means that $q_i - P(c_i, q_i) = S(v_i)$. Condition (20) is thus satisfied with the bidding function S. Hence, to each symmetric BNE under PR we can associate a symmetric BNE in SA.

Now assume that β satisfies (20) and define $P(c_i, q_i) = q_i - \beta(q_i - c_i)$; we verify that P satisfies (8). We know that (9) can be written as $\max_{b_i}(v_i - b_i)\Pr\{b_i > \beta(v_j) \text{ for } j \neq i\}$, and $b_i = \beta(v_i)$ solves this problem because (20) holds. Thus, $p_i = P(c_i, q_i)$ as defined above solves (9). Hence, to each symmetric BNE in SA we can associate a BNE in PR.

Step 2 The unique symmetric BNE in PR is (11).

Proof Step 1 establishes a one-to-one correspondence between symmetric BNE in PR and in SA. Since we know that $\beta(v_i) = v_i - \int_{\underline{v}}^{v_i} \frac{G(t)}{G(v_i)}dt$ is the unique symmetric BNE in SA (in which $\underline{v}$ is the lowest possible valuation), it follows that the unique symmetric BNE under PR is such that $P(c_i, q_i) = q_i - [q_i - c_i - \int_{\underline{q}-c_H}^{q_i-c_i} \frac{G(t)}{G(q_i-c_i)}dt]$, as stated in (11).

In each state of the world, B selects the supplier which offers the highest score, that is he selects supplier i if and only if $q_i - P(c_i, q_i) > q_j - P(c_j, q_j)$. Given (11), this inequality is equivalent to $\beta(q_i - c_i) > \beta(q_j - c_j)$, which reduces to $q_i - c_i = v_i > q_j - c_j = v_j$ since β is strictly increasing. The expected payoff of B is equal to the payoff of a seller in SA, that is the expected second highest valuation in view of the revenue equivalence theorem.

6.3 Proof of Corollary 1

Let $\tilde{G}(v) \equiv G^2(v) + 2G(v)[1 - G(v)] = 1 - [1 - G(v)]^2$ denote the c.d.f. of $\min\{v_1, v_2\}$. Then $E[\min\{v_1, v_2\}] = \int_{\underline{q}-c_H}^{\bar{q}-c_L} v d\tilde{G}(v)$ and in case that $\sigma \leq \theta$ we find

$$\begin{aligned}
E[\min\{v_1, v_2\}] &= \int_{\underline{q}-c_H}^{\bar{q}-c_L} v d\tilde{G}(v) = [v\tilde{G}(v)]_{\underline{q}-c_H}^{\bar{q}-c_L} - \int_{\underline{q}-c_H}^{\bar{q}-c_L} \tilde{G}(v)dv \\
&= \underline{q} + \theta - c_L - \int_{\underline{q}-c_L-\sigma}^{\underline{q}-c_L} \left(1 - (1 - (1-\lambda)\frac{v + c_L + \sigma - \underline{q}}{\theta})^2\right) dv \\
&\quad - \int_{\underline{q}-c_L}^{\underline{q}+\theta-c_L-\sigma} \left(1 - (1 - (1-\lambda)\frac{v + c_L + \sigma - \underline{q}}{\theta} - \lambda\frac{v + c_L - \underline{q}}{\theta})^2\right) dv \\
&\quad - \int_{\underline{q}+\theta-c_L-\sigma}^{\underline{q}+\theta-c_L} \left(1 - (1 - (1 - \lambda + \lambda\frac{v + c_L - \underline{q}}{\theta}))^2\right) dv \\
&= \underline{q} - c_L + \frac{1}{3}\theta - \frac{1}{3}(1-\lambda)\sigma\frac{3\lambda\theta\sigma + 3\theta^2 - \lambda\sigma^2}{\theta^2} = \underline{q} - c_L + \frac{1}{3}[1 - (1-\lambda)(3 + 3\lambda\omega - \lambda\omega^2)\omega]\theta
\end{aligned}$$

Suppliers' rents are

$$U_{PR}^S = 2\int_{\underline{q}-c_H}^{\bar{q}-c_L} G(v)[1-G(v)]dv = \frac{1}{3}\theta + \frac{2}{3}\lambda(1-\lambda)\frac{\sigma^2(3\theta-\sigma)}{\theta^2} = \frac{1}{3}[1+2\lambda(1-\lambda)(3-\omega)\omega^2]\theta$$

In case that $\sigma > \theta$ we find

$$\begin{aligned} E[\min\{v_1,v_2\}] &= \int_{\underline{q}-c_H}^{\bar{q}-c_L} v d\tilde{G}(v) = [v\tilde{G}(v)]_{\underline{q}-c_H}^{\bar{q}-c_L} - \int_{\underline{q}-c_H}^{\bar{q}-c_L} \tilde{G}(v)dv \\ &= \underline{q}+\theta-c_L - \int_{\underline{q}-c_L-\sigma}^{\underline{q}+\theta-c_L-\sigma} \left(1-(1-(1-\lambda)\frac{v+c_L+\sigma-\underline{q}}{\theta})^2\right)dv \\ &\quad - \int_{\underline{q}+\theta-c_L-\sigma}^{\underline{q}-c_L} (1-(1-(1-\lambda))^2)dv - \int_{\underline{q}-c_L}^{\underline{q}+\theta-c_L} \left(1-(1-(1-\lambda+\lambda\frac{v+c_L-\underline{q}}{\theta}))^2\right)dv \\ &= \underline{q}-c_L+\frac{1}{3}(1+\lambda-\lambda^2)\theta-(1-\lambda^2)\sigma = \underline{q}-c_L+\frac{1}{3}[1+\lambda-\lambda^2-3(1-\lambda^2)\omega]\theta \end{aligned}$$

Suppliers' rents are

$$U_{PR}^S = 2\int_{\underline{q}-c_L-\sigma}^{\bar{q}-c_L} G(v)[1-G(v)]dv = \frac{1}{3}(1-2\lambda+2\lambda^2)\theta + 2\lambda(1-\lambda)\sigma = [\frac{1}{3}(1-2\lambda+2\lambda^2)+2\lambda(1-\lambda)\omega]\theta$$

6.4 Proof of Proposition 3

We first notice that for given q_1, q_2, the first score auction we are considering is equivalent to a standard first price auction with two bidders in which the bidders' values are asymmetrically (and independently) distributed such that the set of possible values for bidder 1 is $\{q_1-c_H, q_1-c_L\}$, the set of possible values for bidder 2 is $\{q_2-c_H, q_2-c_L\}$ and $\Pr\{v_1=q_1-c_L\}=\lambda=\Pr\{v_2=q_2-c_L\}$. Precisely, consider supplier i_k, which wins if and only if his opponent j_h bids p_{jh} such that $q_i-p_{ik} > q_j-p_{jh}$, and in this case i_k earns a profit equal to $p_{ik}-c_k$. Let $v_{ik}=q_i-c_k$ and $s_{ik}=q_i-p_{ik}$. Then supplier i_k wins if and only if $s_{ik} > s_{jh}$, and in such a case his payoff is $v_{ik}-s_{ik}$, just like in the standard first price auction in which s_{ik} denotes the monetary bid of bidder i_k. Therefore there is a one-to-one correspondence between BNE in the two settings, and in Proposition 1 in Doni and Menicucci (2011) we prove that a unique equilibrium outcome exists in a standard first price auction; thus we obtain the same result for the first score auction under policy PU.[22]

We prove in the following that the strategy profiles described by Proposition 3 are indeed BNE. For each supplier i_k, given the bids of the types of supplier j, let $w_i(p_{ik})$ and $\pi_{ik}(p_{ik})$ denote respectively his probability of winning (which does not depend on the type k) and his payoff when bidding p_{ik}.

(i) **Type 1_L and 1_H**. Both type 2_L and type 2_H offer a score of q_1-c_L. Therefore the payoff of 1_L is zero if he offers as specified by Proposition 3(i), and in order to win he needs to bid below c_L. This yields a negative payoff in case of victory, and therefore his strategy in Proposition 3(i) is a best reply. A very similar argument applies for type 1_H.

Type 2_H. Given the strategies in Proposition 3(i), the payoff of 2_H is $t+c_L-c_H$ (notice that $t+c_L-c_H>0$ since $\lambda t>\sigma$). It is obviously unprofitable for him to bid below $t+c_L$, since then he still wins with certainty but his revenue is smaller. Regarding bids above $t+c_L$, the strategies of 1_L and 1_H need to be such that no $p>t+c_L$ is profitable. We prove that this is the case if, for instance, Φ_{1L} is the

[22] Strictly speaking, Proposition 1 in Doni and Menicucci (2011) is stated with reference to a tie-breaking rule which is different fromt the tie-breaking rule considered in this paper, but in fact the tie-breaking rule is irrelevant for the case in which the distribution of values of one bidder is obtained by shifting the distribution of values of the other bidder.

uniform distribution on $[c_L, \alpha c_L]$ and Φ_{1H} is the uniform distribution on $[c_H, \alpha c_H]$ with $\alpha > 1$ and close to 1. Precisely, if 2_H bids $p_{2H} \in (t+c_L, t+\alpha c_L]$, then $w_2(p_{2H}) = 1-\lambda+\lambda(1-\Pr\{p_{1L} < p_{2H}-t\}) = 1-\lambda\frac{p_{2H}-t-c_L}{(\alpha-1)c_L}$ and $\pi_{2H}(p_{2H}) = (p_{2H}-c_H)[1-\frac{\lambda(p_{2H}-t-c_L)}{(\alpha-1)c_L}]$ and it is simple to see that π_{2H} is decreasing in $(t+c_L, t+\alpha c_L]$ for α close to 1. For bids in $(t+\alpha c_L, t+c_H]$, the resulting score is between $q_1 - c_H$ and $q_1 - \alpha c_L$, thus $w_2(p_{2H}) = 1-\lambda$ and $\pi_{2H}(p_{2H}) = (1-\lambda)(p_{2H}-c_H)$ for $p_{2H} \in (t+\alpha c_L, t+c_H]$; thus π_{2H} is increasing in this interval, with $\pi_{2H}(t+c_H) = (1-\lambda)t$, which is smaller than $t-\sigma$ since $\lambda t \geq \sigma$. Finally, if 2_H bids $p_{2H} \in (t+c_H, t+\alpha c_H]$, then $w_2(p_{2H}) = (1-\lambda)(1-\Pr\{p_{1H} < p_{2H}-t\}) = (1-\lambda)(1-\frac{p_{2H}-t-c_H}{(\alpha-1)c_H})$; thus $\pi_{2H}(p_{2H}) = (1-\lambda)(p_{2H}-c_H)[1-\frac{p_{2H}-t-c_H}{(\alpha-1)c_H}]$, and it is simple to see that π_{2H} is decreasing in $(t+c_H, t+\alpha c_H]$ for α close to 1.

Type 2_L. Given the strategies in Proposition 3(i), the payoff of 2_L is t. Thus we need to prove that $(p_{2L}-c_L)w_2(p_{2L}) \leq t$ for any $p_{2L} \geq t+c_L$ When arguing about type 2_H we have proved that $(p-c_H)w_2(p) \leq t+c_L-c_H$ for any $p > t+c_L$, and this property can be written as $(p-c_L)w_2(p)+\sigma \leq t+\sigma w_2(p)$ for any $p > t+c_L$, which implies $(p-c_L)w_2(p) \leq t$ for any $p \geq t+c_L$.

(ii) Let $\tilde{p}$ be the largest solution to $h(p) \equiv (1-\lambda)p^2 - [(1-2\lambda)t+(1-\lambda)(c_H+c_L)]p+(1-\lambda)(t+c_H)c_L - \lambda t c_H = 0$, that is

$$\tilde{p} = c_L + \frac{1}{2}\sigma + \frac{1-2\lambda}{2(1-\lambda)}t + \frac{1}{2(1-\lambda)}\sqrt{[t+(1-\lambda)\sigma]^2 - 4\lambda(1-\lambda)t^2} \tag{21}$$

and notice that $h(t+c_L) = t(\lambda t - \sigma) < 0$, $h(c_H) = -(1-\lambda)t\sigma < 0$, $h(t+c_H) = \lambda t^2 > 0$. This implies that $\max\{t+c_L, c_H\} < \tilde{p} < t+c_H$ and $t+c_L < \hat{p} < \tilde{p}$, given $\hat{p} = (1-\lambda)\tilde{p}+\lambda(t+c_L)$.

Type 1_H. Given Φ_{2L}, Φ_{2H}, each type of supplier 2 offers a score larger or equal to $q_1 - c_H$. Hence, the same argument given for type 1_H in the proof of Proposition 3(i) applies here.

Type 1_L. Given Φ_{2L}, Φ_{2H}, we show that $\pi_{1L}(p_{1L}) = (1-\lambda)(\tilde{p}-t-c_L) > 0$ for any $p_{1L} \in [\hat{p}-t, c_H)$, and $\pi_{1L}(p_{1L}) < (1-\lambda)(\tilde{p}-t-c_L)$ if $p_{1L} \notin [\hat{p}-t, c_H)$.
If $p_{1L} \in [\hat{p}-t, \tilde{p}-t)$, then the score offered by 1_L belongs to $(q_2-\tilde{p}, q_2-\hat{p}]$ and $w_1(p_{1L}) = 1-\lambda+\lambda[1-\Phi_{2L}(t+p_{1L})] = (1-\lambda)\frac{\tilde{p}-t-c_L}{p_{1L}-c_L}$, $\pi_{1L}(p_{1L}) = (1-\lambda)(\tilde{p}-t-c_L)$. Likewise, if $p_{1L} \in [\tilde{p}-t, c_H)$ (which corresponds to a score in $(q_1-c_H, q_2-\tilde{p}]$) then $w_1(p_{1L}) = (1-\lambda)[1-\Phi_{2H}(t+p_{1L})] = (1-\lambda)\frac{\tilde{p}-t-c_L}{p_{1L}-c_L}$ and $\pi_{1L}(p_{1L}) = (1-\lambda)(\tilde{p}-t-c_L)$. Hence, $\pi_{1L}(p_{1L}) = (1-\lambda)(\tilde{p}-t-c_L) > 0$ for any $p_{1L} \in [\hat{p}-t, c_H)$.
No $p_{1L} \notin [\hat{p}-t, c_H)$ is profitable for 1_L since (i) if $p_{1L} > c_H$, then $w_1(p_{1L}) = 0$ and $\pi_{1L}(p_{1L}) = 0$; (ii) if $p_{1L} < \hat{p}-t$, then $w_1(p_{1L}) = 1$ (since his score is larger than $q_2-\hat{p}$) but $\pi_{1L}(p_{1L}) < \hat{p}-t-c_L = (1-\lambda)(\tilde{p}-t-c_L)$; (iii) if $p_{1L} = c_H$, then 1_L may win only tying with type 2_H, which occurs with probability $(1-\lambda)(1-\frac{t+c_H-\tilde{p}}{\sigma}) = (1-\lambda)\frac{\tilde{p}-t-c_L}{\sigma}$ (see Φ_{2H}), and thus $w_1(c_H) = (1-\lambda)\frac{\tilde{p}-t-c_L}{2\sigma}$, $\pi_{1H}(c_H) = (1-\lambda)\frac{\tilde{p}-t-c_L}{2} < (1-\lambda)(\tilde{p}-t-c_L)$.

Type 2_H. Given Φ_{1L}, we show that $\pi_{2H}(p_{2H}) = (1-\lambda)t$ for any $p_{2H} \in [\tilde{p}, t+c_H]$ and $\pi_{2H}(p_{2H}) < (1-\lambda)t$ for $p_{2H} \notin [\tilde{p}, t+c_H]$.
If $p_{2H} \in [\tilde{p}, t+c_H]$ (which corresponds to a score in $[q_1-c_H, q_2-\tilde{p}]$), then $w_2(p_{2H}) = 1-\lambda+\lambda[1-\Phi_{1L}(p_2-t)] = \frac{(1-\lambda)t}{p_2-c_H}$ and $\pi_{2H}(p_{2H}) = (1-\lambda)t$.
No $p_{2H} \notin [\tilde{p}, t+c_H]$ is profitable for 2_H since (i) Φ_{1H} is such that type 2_H has no incentive to bid above $t+c_H$ (for instance, this is the case if Φ_{1H} is the uniform distribution on $[c_H, \alpha c_H]$ with $\alpha > 1$ and α close to 1); (ii) if $p_{2H} \in [\hat{p}, \tilde{p})$ (which corresponds to a score in $(q_2-\tilde{p}, q_2-\hat{p}]$), then $w_2(p_{2H}) = 1-\lambda+\lambda[1-\Phi_{1L}(p_2-t)] = \frac{\hat{p}-c_L}{p_{2H}-c_L}$ and $\pi_{2H}(p_{2H}) = \frac{(p_{2H}-c_H)(\hat{p}-c_L)}{p_{2H}-c_L}$, which is increasing in p_{2H}; (iii) if $p_{2H} < \hat{p}$, then $w_2(p_{2H}) = 1$ and $\pi_{2H}(p_{2H}) = p_{2H}-c_H < \hat{p}-c_H = \pi_{2H}(\hat{p})$.

Type 2_L. Given Φ_{1L}, we show that $\pi_{2L}(p_{2L}) = \hat{p}-c_L$ for any $p_{2L} \in [\hat{p}, \tilde{p}]$ and $\pi_{2L}(p_{2L}) < \hat{p}-c_L$ for any $p_{2L} \notin [\hat{p}, \tilde{p}]$.

We already know that $w_2(p_{2L}) = \frac{\hat{p}-c_L}{p_{2L}-c_L}$ if $p_{2L} \in [\hat{p}, \tilde{p}]$ and $w_2(p_{2L}) = \frac{(1-\lambda)t}{p_2-c_H}$ if $p_{2L} \in (\tilde{p}, t+c_H]$. Therefore $\pi_{2L}(p_{2L}) = \hat{p} - c_L$ if $p_{2L} \in [\hat{p}, \tilde{p}]$ and $\pi_{2L}(p_{2L}) = (1-\lambda)t\frac{p_{2L}-c_L}{p_{2L}-c_H}$ if $p_{2L} \in (\tilde{p}, t+c_H]$, which is decreasing in p_{2L}. No bid below $\hat{p}$ is profitable for 2_L since $w_2(p_{2L}) = 1$ and $\pi_{2L}(p_{2L}) = p_{2L} - c_L < \hat{p} - c_L$ for any $p_{2L} < \hat{p}$. Finally, no bid above $t + c_H$ is profitable since Φ_{1H} is such that type 2_L has no incentive to bid above $t + c_H$ (for instance, this is the case if Φ_{1H} is the uniform distribution on $[c_H, \alpha c_H]$ with $\alpha > 1$ and α close to 1).

Evaluation of $\Pr\{1_L$ **def** $2_L\}$ Notice that

$$\Pr\{1_L \text{ def } 2_L\} = \Pr\{q_1 - p_{1L} > q_2 - p_{2L}\} = \Pr\{p_{2L} > t + p_{1L}\} = \int_{\hat{p}-t}^{\tilde{p}-t} \Phi'_{1L}(p_1)[1 - \Phi_{2L}(t+p_1)]dp_1$$

and since

$$\Phi'_{1L}(p) = \begin{cases} \frac{\hat{p}-c_L}{\lambda(p+t-c_L)^2} & \text{for } p \in [\hat{p}-t, \tilde{p}-t) \\ \frac{1-\lambda}{\lambda}\frac{t}{(p+t-c_H)^2} & \text{for } p \in (\tilde{p}-t, c_H) \end{cases} \tag{22}$$

we obtain

$$\Pr\{1_L \text{ def } 2_L\} = \int_{\hat{p}-t}^{\tilde{p}-t} \frac{\hat{p}-c_L}{\lambda(p_1+t-c_L)^2}\frac{1-\lambda}{\lambda}\frac{\tilde{p}-t-p_1}{p_1-c_L}dp_1 = \frac{(\hat{p}-c_L)(1-\lambda)}{\lambda^2}\int_{\hat{p}-t}^{\tilde{p}-t}\frac{\tilde{p}-t-p_1}{(p_1+t-c_L)^2(p_1-c_L)}dp_1$$

We exploit

$$\int \frac{\tilde{p}-t-p_1}{(p_1+t-c_L)^2(p_1-c_L)}dp_1 = \frac{\tilde{p}-c_L-t}{t^2}\ln\left|\frac{p_1-c_L}{p_1-c_L+t}\right| + \frac{\tilde{p}-c_L}{t(p_1-c_L+t)}$$

to obtain

$$\int_{\hat{p}-t}^{\tilde{p}-t}\frac{\tilde{p}-t-p_1}{(p_1+t-c_L)^2(p_1-c_L)}dp_1 = \frac{\tilde{p}-c_L-t}{t^2}\ln\left|\frac{(\tilde{p}-t-c_L)(\hat{p}-c_L)}{(\tilde{p}-c_L)(\hat{p}-t-c_L)}\right| - \frac{\tilde{p}-\hat{p}}{t\,(\hat{p}-c_L)}$$

and finally

$$\begin{aligned} \Pr\{1_L \text{ def } 2_L\} &= \frac{(\hat{p}-c_L)(1-\lambda)}{\lambda^2}\int_{\hat{p}-t}^{\tilde{p}-t}\frac{\tilde{p}-t-p_1}{(p_1+t-c_L)^2(p_1-c_L)}dp_1 \\ &= \frac{(\hat{p}-c_L)(1-\lambda)(\tilde{p}-c_L-t)}{\lambda^2 t^2}\ln\left|\frac{(\tilde{p}-t-c_L)(\hat{p}-c_L)}{(\tilde{p}-c_L)(\hat{p}-t-c_L)}\right| - \frac{(1-\lambda)(\tilde{p}-\hat{p})}{\lambda^2 t} \end{aligned}$$

Evaluation of $\Pr\{1_L$ **def** $2_H\}$ Notice that

$$\begin{aligned} \Pr\{1_L \text{ def } 2_H\} &= \Pr\{q_1 - p_{1L} > q_2 - p_{2H}\} = \Pr\{p_{2H} > t + p_{1L}\} \\ &= \Phi_{1L}(\tilde{p}-t) + \int_{\tilde{p}-t}^{c_H}\Phi'_{1L}(p_1)[1-\Phi_{2H}(t+p_1)]dp_1 \end{aligned}$$

and using again (22) we obtain

$$\begin{aligned} \Pr\{1_L \text{ def } 2_H\} &= \frac{1}{\lambda}\frac{\tilde{p}-\hat{p}}{\tilde{p}-c_L} + \int_{\tilde{p}-t}^{c_H}\frac{1-\lambda}{\lambda}\frac{t}{(p_1+t-c_H)^2}(1-\frac{t+p_1-\tilde{p}}{p_1-c_L})dp_1 \\ &= \frac{1}{\lambda}\frac{\tilde{p}-\hat{p}}{\tilde{p}-c_L} + \frac{(1-\lambda)t(\tilde{p}-c_L-t)}{\lambda}\int_{\tilde{p}-t}^{c_H}\frac{1}{(p_1+t-c_H)^2(p_1-c_L)}dp_1 \end{aligned}$$

We exploit

$$\int \frac{1}{(p_1+t-c_H)^2(p_1-c_L)}dp_1 = \frac{1}{(t-\sigma)^2}\ln\left|\frac{p_1-c_L}{p_1+t-c_H}\right| + \frac{1}{(t-\sigma)\,(p_1+t-c_H)}$$

to obtain

$$\int_{\tilde{p}-t}^{c_H} \frac{1}{(p_1+t-c_H)^2(p_1-c_L)}dp_1 = \frac{1}{(t-\sigma)^2}\ln\frac{\sigma(\tilde{p}-c_H)}{t(\tilde{p}-t-c_L)} + \frac{\tilde{p}-c_H-t}{(t-\sigma)\,t\,(\tilde{p}-c_H)}$$

and finally

$$\begin{aligned}\Pr\{1_L \text{ def } 2_H\} &= \frac{1}{\lambda}\frac{\tilde{p}-\hat{p}}{\tilde{p}-c_L} + \frac{(1-\lambda)t(\tilde{p}-c_L-t)}{\lambda}\int_{\tilde{p}-t}^{c_H}\frac{1}{(p_1+t-c_H)^2(p_1-c_L)}dp_1 \\ &= \frac{1}{\lambda}\frac{\tilde{p}-\hat{p}}{\tilde{p}-c_L} + \frac{(1-\lambda)t(\tilde{p}-c_L-t)}{\lambda\,(t-\sigma)^2}\ln\frac{\sigma(\tilde{p}-c_H)}{t(\tilde{p}-t-c_L)} + \frac{(1-\lambda)(\tilde{p}-c_L-t)(\tilde{p}-c_H-t)}{\lambda\,(t-\sigma)\,(\tilde{p}-c_H)}\end{aligned}$$

6.5 Proof of Corollary 2

We have explained in the text that $u^B_{PU}(q_1,q_2) = q_1 - c_L$ when $\lambda t \geq \sigma$. When $\lambda t < \sigma$ we can evaluate $u^B_{PU}(q_1,q_2)$ as the difference between the social surplus $ss_{PU}(q_1,q_2)$ and the suppliers' payoff $u^S_{PU}(q_1,q_2)$: $u^B_{PU}(q_1,q_2) = ss_{PU}(q_1,q_2) - u^S_{PU}(q_1,q_2)$. Precisely,

$$\begin{aligned}ss_{PU}(q_1,q_2) &= \lambda^2(q_2-c_L-t\Pr\{1_L \text{ def } 2_L\}) + \lambda(1-\lambda)(q_2-c_H-(t-\sigma)\Pr\{1_L \text{ def } 2_H\}) \\ &\quad +(1-\lambda)(q_2-\lambda c_L-(1-\lambda)c_H) \\ &= q_2-c_L-(1-\lambda)\sigma-\lambda^2 t\Pr\{1_L \text{ def } 2_L\}-\lambda(1-\lambda)(t-\sigma)\Pr\{1_L \text{ def } 2_H\}\end{aligned}$$

Moreover, in the proof of Proposition 3(ii) we see that the rents of types 1_L, 2_L, 2_H are $(1-\lambda)(\tilde{p}-t-c_L)$, $\hat{p}-c_L$, $(1-\lambda)t$, respectively. Hence

$$\begin{aligned}u^S_{PU}(q_1,q_2) &= \lambda(1-\lambda)(\tilde{p}-t-c_L)+\lambda(\hat{p}-c_L)+(1-\lambda)^2 t \\ &= (1-\lambda)^2\,t+\lambda\,(1-\lambda)\,\sigma+\lambda\sqrt{[t+(1-\lambda)\,\sigma]^2-4\lambda\,(1-\lambda)\,t^2}\end{aligned}$$

in which the second equality is obtained using $\hat{p} = (1-\lambda)\tilde{p}+\lambda(t+c_L)$ and (21). From $ss_{PU}(q_1,q_2) - u^S_{PU}(q_1,q_2)$ we obtain the expression of $u^B_{PU}(q_1,q_2)$ provided by Corollary 2 for the case of $\lambda t < \sigma$.

6.6 Proof of Proposition 4

(ii) In view of Propositions 1(ii) and Corollary1(i), we find that $U^B_C - U^B_{PR} = 2\lambda(1-\lambda)\sigma - \frac{1}{6}(2\lambda^2-14\lambda+13)\theta$, which yields the result.

(i) The proof is organized in 9 steps.

Step 1 There exists a unique solution to $\Psi(x) = 0$ [i.e., equation (18)] in $(0,1)$, which we denote with $\hat{x}$, and $\Psi(x) < 0$ for $x \in [0,\hat{x})$, $\Psi(x) > 0$ for $x \in (\hat{x},1]$.
Proof The proof is found in proof of Proposition 1.

Step 2 Let $\omega = \sigma/\theta$. When $\sigma \leq \theta$, the inequality $U^B_{PR} > U^B_C$ is equivalent to $(1-2\lambda)\Gamma(\hat{x}) > 0$ with

$$\begin{aligned}\Gamma(x) &= 2\lambda(1-2\lambda)(1-\lambda)(x-3)x^3+(9-12\lambda+12\lambda^2)x^2 \\ &\quad -\left(2\omega^2\lambda(1-2\lambda)(1-\lambda)(3-\omega)+(12\lambda^2-12\lambda+6)\omega+14-4\lambda\right)x+6\omega\end{aligned}$$

Proof When $\sigma \leq \theta$, the difference between the payoff of the buyer under PR and his payoff under C is

$$\begin{aligned} U_{PR}^B - U_C^B &= (\underline{q} - c_L + \frac{1}{3}\theta - (1-\lambda)\omega(1+\lambda\omega - \frac{1}{3}\lambda\omega^2)\theta) - (\underline{q} - p_L + \frac{2}{3}\theta - (1-\lambda)(1-\lambda\hat{x} + \frac{1}{3}\lambda\hat{x}^2)\hat{\Delta}) \quad (23) \\ &= (1-\lambda)\hat{x}(1-\lambda\hat{x}+\frac{1}{3}\lambda\hat{x}^2)\theta + \frac{(1+2\lambda)\hat{x} - 2(2+\lambda\omega) + 2\frac{\omega}{\hat{x}}}{2(1-2\lambda)}\theta - (1-\lambda)\omega(1+\lambda\omega - \frac{1}{3}\lambda\omega^2)\theta - \frac{1}{3}\theta \\ &= \frac{\theta}{6\hat{x}(1-2\lambda)^2}(1-2\lambda)\Gamma(\hat{x}) \end{aligned}$$

Step 3 There exists a unique solution to $\Gamma(x) = 0$ in $(0,1)$, which we denote with x', and $\Gamma(x) > 0$ for $x \in (0, x')$, $\Gamma(x) < 0$ for $x \in (x', 1)$.

Proof We find that $\Gamma(0) = 6\omega > 0$ and $\Gamma(1) = 2\lambda(2\lambda-1)(\lambda-1)\omega^3 - 6\lambda(2\lambda-1)(\lambda-1)\omega^2 + 12\lambda(1-\lambda)\omega - 12\lambda + 24\lambda^2 - 8\lambda^3 - 5$; we prove that $v_\lambda(\omega) \equiv \Gamma(1)$ is negative, which implies that a solution to $\Gamma(x) = 0$ exists in $(0,1)$. Since $v''_\lambda(\omega) = 12\lambda(2\lambda-1)(1-\lambda)(1-\omega)$, it follows that $v''_\lambda(\omega) \geq 0$ if $\lambda \geq \frac{1}{2}$; then from $v_\lambda(0) = -\lambda^2 - (1-\lambda)(5+17\lambda-8\lambda^2) < 0$, $v_\lambda(1) = -\lambda^3 - (1-\lambda)(5+9\lambda-15\lambda^2) < 0$ we infer that $v_\lambda(\omega) < 0$ for any $\omega \in [0,1]$ if $\lambda \geq \frac{1}{2}$. For $\lambda < \frac{1}{2}$, we see that $v_\lambda(\omega) < 0$ for any $\omega \in [0,1]$ since $v'_\lambda(\omega) = 6\lambda(1-\lambda)(2-2\omega+\omega^2-2\lambda\omega^2+4\lambda\omega)$ is positive for any $\omega \in (0,1)$, and $v_\lambda(1) < 0$ (mentioned above).

We now show that Γ is strictly convex; this implies that the solution x' to $\Gamma(x) = 0$ is unique and that $\Gamma(x) > 0$ for $x \in (0,x')$, $\Gamma(x) < 0$ for $x \in (x',1)$. Indeed, $\Gamma''(x) = 18 - 24\lambda + 24\lambda^2 + 12\lambda(2\lambda-1)(1-\lambda)(3x - x^2)$, and since $18 - 24\lambda + 24\lambda^2 > 0$ for any $\lambda \in (0,1)$ we infer that $\Gamma''(x) > 0$ for $\lambda \geq \frac{1}{2}$. If instead $\lambda < \frac{1}{2}$, then we exploit $3x - 2x^2 \leq \frac{3}{2}$ for $x \in (0,1)$ and thus $\Gamma''(x) \geq 18 - 24\lambda + 24\lambda^2 + 12\lambda(2\lambda-1)(1-\lambda)\frac{3}{2} = 6(3 - 7\lambda + 13\lambda^2 - 6\lambda^3) > 0$.

Step 4 If $\xi(\omega,\lambda)$ is such that $\Psi[\xi(\omega,\lambda)] > 0$ and $\Gamma[\xi(\omega,\lambda)] > 0$ for any $\omega \in (0,1]$ and any $\lambda \in (0,\frac{1}{2})$, then $U_{PR}^B > U_C^B$ for any $\omega \in (0,1]$ and any $\lambda \in (0,\frac{1}{2})$.

Proof From $\Psi[\xi(\omega,\lambda)] > 0$ and Step 1 we infer that $\hat{x} < \xi(\omega,\lambda)$. From $\Gamma[\xi(\omega,\lambda)] > 0$ and Step 3 we infer that $\Gamma(\hat{x}) > 0$, thus $U_{PR}^B > U_C^B$ since $\lambda < \frac{1}{2}$.

Step 5 Let $\xi(\omega,\lambda) \equiv \begin{cases} \frac{2}{5}\omega + \frac{4}{25}\omega\lambda + \frac{2}{25}\omega\lambda^2 & \text{for } \omega \in (0, \frac{5}{12}] \\ \frac{3}{8}\omega + \frac{1}{5}\omega\lambda + \frac{1}{10}\omega\lambda^2 & \text{for } \omega \in (\frac{5}{12}, \frac{7}{10}] \\ \frac{9}{25}\omega + \frac{17}{75}\omega\lambda + \frac{8}{75}\omega\lambda^2 & \text{for } \omega \in (\frac{7}{10}, \frac{5}{6}] \\ \frac{7}{20}\omega + \frac{9}{40}\omega\lambda + \frac{3}{20}\omega\lambda^2 & \text{for } \omega \in (\frac{5}{6}, 1] \end{cases}$. Then $\Psi[\xi(\omega,\lambda)] > 0$ for any $\omega \in (0,1]$ and any $\lambda \in (0,\frac{1}{2})$.

Proof Using $\xi(\omega,\lambda)$ we obtain

$$\Psi[\xi(\omega,\lambda)] = \begin{cases} \frac{2\omega(1-2\lambda)}{15625}\psi_{\lambda 1}(\omega) & \text{for } \omega \in (0, \frac{5}{12}] \\ \frac{\omega(1-2\lambda)}{16000}\psi_{\lambda 2}(\omega) & \text{for } \omega \in (\frac{5}{12}, \frac{7}{10}] \\ \frac{\omega(1-2\lambda)}{421875}\psi_{\lambda 3}(\omega) & \text{for } \omega \in (\frac{7}{10}, \frac{5}{6}] \\ \frac{\omega(1-2\lambda)}{16000}\psi_{\lambda 4}(\omega) & \text{for } \omega \in (\frac{5}{6}, 1] \end{cases}.$$

with

$$
\begin{aligned}
\psi_{\lambda 1}(\omega) &= -4\lambda\left(2\lambda+5\right)\left(1-\lambda\right)\left(\lambda^2+2\lambda+5\right)^2\omega^2-50(\lambda^2+2\lambda+5)(2\lambda^3+3\lambda^2+6\lambda-10)\omega+625\lambda^2 \\
\psi_{\lambda 2}(\omega) &= -\lambda(2\lambda+5)(1-\lambda)(4\lambda^2+8\lambda+15)^2\omega^2-10(4\lambda^2+8\lambda+15)(8\lambda^3+12\lambda^2+14\lambda-35)\omega-400(5-4\lambda^2) \\
\psi_{\lambda 3}(\omega) &= \begin{pmatrix} -2\lambda\left(8\lambda+21\right)\left(1-\lambda\right)\left(27+17\lambda+8\lambda^2\right)^2\omega^2 \\ -75\left(27+17\lambda+8\lambda^2\right)\left(16\lambda^3+26\lambda^2+21\lambda-69\right)\omega+625(72\lambda^2-135+9\lambda) \end{pmatrix} \\
\psi_{\lambda 4}(\omega) &= \begin{pmatrix} -3\lambda(\lambda+2)(1-\lambda)(14+9\lambda+6\lambda^2)^2\omega^2 \\ -10(14+9\lambda+6\lambda^2)(12\lambda^3+12\lambda^2+7\lambda-38)\omega-400(10+3\lambda-6\lambda^2) \end{pmatrix}
\end{aligned}
$$

Now we prove that $\psi_{\lambda 1}(\omega)>0$ for any $\omega\in[0,\frac{5}{12}]$, $\psi_{\lambda 2}(\omega)>0$ for any $\omega\in[\frac{5}{12},\frac{7}{10}]$, $\psi_{\lambda 3}(\omega)>0$ for any $\omega\in[\frac{7}{10},\frac{5}{6}]$, and $\psi_{\lambda 4}(\omega)>0$ for any $\omega\in[\frac{5}{6},1]$. The proof relies on the fact that, for $i=1,2,3,4$, $\psi_{\lambda i}$ is concave and is positive at the extreme points of the interval to which $\psi_{\lambda i}$ refers.

- $\psi_{\lambda 1}(\omega)>0$ for any $\omega\in[0,\frac{5}{12}]$ since $\psi_{\lambda 1}(0)=625\lambda^2>0$, $\psi_{\lambda 1}(\frac{5}{12})=\frac{3125}{3}-\frac{10625}{36}\lambda-\frac{3875}{9}\lambda^3-\frac{925}{9}\lambda^4-\frac{625}{36}\lambda^5+\frac{9125}{36}\lambda^2+\frac{275}{36}\lambda^6+\frac{25}{18}\lambda^7>\frac{3125}{3}-\frac{10625}{36}(\frac{1}{2})-\frac{3875}{9}(\frac{1}{2})^3-\frac{925}{9}(\frac{1}{2})^4-\frac{625}{36}(\frac{1}{2})^5=\frac{959975}{1152}>0$ for any $\lambda\in(0,\frac{1}{2})$.
- $\psi_{\lambda 2}(\omega)>0$ for any $\omega\in[\frac{5}{12},\frac{7}{10}]$ since $\psi_{\lambda 2}(\frac{5}{12})=\frac{375}{2}-\frac{78475}{72}\lambda^3-\frac{6175}{18}\lambda^4-50\lambda^5+\frac{4625}{48}\lambda+\frac{42025}{48}\lambda^2+\frac{275}{9}\lambda^6+\frac{50}{9}\lambda^7>\frac{375}{2}-\frac{78475}{72}(\frac{1}{2})^3-\frac{6175}{18}(\frac{1}{2})^4-50(\frac{1}{2})^5=\frac{5425}{192}>0$, $\psi_{\lambda 2}(\frac{7}{10})=1675-\frac{245}{4}\lambda-\frac{3563}{2}\lambda^3-\frac{10878}{25}\lambda^4+\frac{1115}{4}\lambda^2+\frac{56}{5}\lambda^5+\frac{2156}{25}\lambda^6+\frac{392}{25}\lambda^7>1675-\frac{245}{4}(\frac{1}{2})-\frac{3563}{2}(\frac{1}{2})^3-\frac{10878}{25}(\frac{1}{2})^4=\frac{557797}{400}>0$ for any $\lambda\in(0,\frac{1}{2})$.
- $\psi_{\lambda 3}(\omega)>0$ for any $\omega\in[\frac{7}{10},\frac{5}{6}]$ since $\psi_{\lambda 3}(\frac{7}{10})=\frac{26865}{2}-\frac{104265}{2}\lambda^3-\frac{144151}{10}\lambda^4+\frac{1121859}{50}\lambda+\frac{219438}{25}\lambda^2+\frac{5404}{5}\lambda^5+\frac{73696}{25}\lambda^6+\frac{12544}{25}\lambda^7>\frac{26865}{2}-\frac{104265}{2}(\frac{1}{2})^3-\frac{144151}{10}(\frac{1}{2})^4=\frac{962399}{160}>0$ and $\psi_{\lambda 3}(\frac{5}{6})=\frac{64125}{2}-300\lambda^2-\frac{368875}{6}\lambda^3-\frac{264875}{18}\lambda^4+\frac{44475}{2}\lambda+\frac{27500}{9}\lambda^5+\frac{37600}{9}\lambda^6+\frac{6400}{9}\lambda^7>\frac{64125}{2}-300(\frac{1}{2})^2-\frac{368875}{6}(\frac{1}{2})^3-\frac{264875}{18}(\frac{1}{2})^4=\frac{6734275}{288}>0$ for any $\lambda\in(0,\frac{1}{2})$.
- $\psi_{\lambda 4}(\omega)>0$ for any $\omega\in[\frac{5}{6},1]$ since $\psi_{\lambda 4}(\frac{5}{6})=\frac{1300}{3}-\frac{16525}{6}\lambda^3-\frac{3625}{4}\lambda^4-\frac{25}{4}\lambda^5+\frac{50}{3}\lambda+\frac{5200}{3}\lambda^2+300\lambda^6+75\lambda^7>\frac{1300}{3}-\frac{16525}{6}(\frac{1}{2})^3-\frac{3625}{4}(\frac{1}{2})^4-\frac{25}{4}(\frac{1}{2})^5=\frac{4125}{128}>0$, $\psi_{\lambda 4}(1)=1320-3330\lambda^3-945\lambda^4+64\lambda+1446\lambda^2+135\lambda^5+432\lambda^6+108\lambda^7>1320-3330(\frac{1}{2})^3-945(\frac{1}{2})^4=\frac{13515}{16}>0$ for any $\lambda\in(0,\frac{1}{2})$.

Step 6 For the same $\xi(\omega,\lambda)$ introduced in Step 5, the inequality $\Gamma[\xi(\omega,\lambda)]>0$ holds for any $\omega\in(0,1]$ and any $\lambda\in(0,\frac{1}{2})$. In view of Steps 4 and 5, we conclude that $U_{PR}^B>U_C^B$ for any $\omega\in(0,1]$ and any $\lambda\in(0,\frac{1}{2})$.
Proof For $\omega\in(0,\frac{5}{12}]$ we have

$$
\Gamma[\xi(\omega,\lambda)]=\frac{2\omega\left(1-2\lambda\right)}{390625}\begin{pmatrix} 2\lambda\left(1-\lambda\right)\left(\lambda^2+2\lambda+5\right)\left(2\lambda^2+4\lambda+35\right)\left(4\lambda^4+16\lambda^3+6\lambda^2-20\lambda+475\right)\omega^3 \\ -150\lambda\left(1-\lambda\right)\left(\lambda^2+2\lambda+5\right)\left(4\lambda^4+16\lambda^3+56\lambda^2+80\lambda+725\right)\omega^2 \\ -3750\left(\lambda^2+2\lambda+5\right)\left(2\lambda^3+3\lambda^2-16\lambda+10\right)\omega+78125+31250\lambda-31250\lambda^2 \end{pmatrix}
$$

We use $\gamma_{\lambda 1}(\omega)$ to denote the term in parenthesis and notice that

$$
\gamma_{\lambda 1}''(\omega)=12\lambda\left(1-\lambda\right)\left(\lambda^2+2\lambda+5\right)\begin{pmatrix} \left(2\lambda^2+4\lambda+35\right)\left(4\lambda^4+16\lambda^3+6\lambda^2-20\lambda+475\right)\omega \\ -2000\lambda-400\lambda^3-18125-100\lambda^4-1400\lambda^2 \end{pmatrix}
$$

is negative for $\omega\in(0,\frac{5}{12}]$.[23] Hence $\gamma_{\lambda 1}$ is concave and $\gamma_{\lambda 1}(\omega)>0$ for any $\omega\in(0,\frac{5}{12}]$ since $\gamma_1(0)=78125+31250\lambda\left(1-\lambda\right)>0$ and $\gamma_{\lambda 1}(\frac{5}{12})=\lambda(\frac{36828125}{864}-\frac{875}{108}\lambda^8-\frac{125}{108}\lambda^9)+\frac{1878125}{96}\lambda^2+\frac{13559375}{864}\lambda^3+\frac{2783125}{288}\lambda^4+\frac{6625}{9}\lambda^5+\frac{51125}{27}\lambda^6+\frac{2375}{6}\lambda^7+\frac{125}{2}\lambda^8>0$ for $\lambda\in(0,\frac{1}{2})$.

[23] $\gamma_{\lambda 1}''$ is increasing and $\gamma_{\lambda 1}''(\frac{5}{12})=-5\lambda(1-\lambda)(\lambda^2+2\lambda+5)(26875+3600\lambda+2280\lambda^2+416\lambda^3+24\lambda^4-48\lambda^5-8\lambda^6)<0$.

For $\omega \in (\frac{5}{12}, \frac{7}{10}]$ we have

$$\Gamma[\xi(\omega,\lambda)] = \frac{\omega(1-2\lambda)}{1280000}\left(\begin{array}{c} \lambda(1-\lambda)\left(4\lambda^2+8\lambda+15\right)\left(4\lambda^2+8\lambda+55\right)\left(16\lambda^4+64\lambda^3+24\lambda^2-80\lambda+1225\right)\omega^3 \\ +120\lambda(\lambda-1)\left(4\lambda^2+8\lambda+15\right)\left(16\lambda^4+64\lambda^3+184\lambda^2+240\lambda+1825\right)\omega^2 \\ -2400\left(4\lambda^2+8\lambda+15\right)\left(8\lambda^3+12\lambda^2-54\lambda+35\right)\omega-64000(2\lambda+3)(2\lambda-5) \end{array}\right)$$

We use $\gamma_{\lambda 2}(\omega)$ to denote the term in parenthesis and notice that

$$\gamma''_{\lambda 2}(\omega) = 6\lambda(1-\lambda)\left(4\lambda^2+8\lambda+15\right)\left(\begin{array}{c} \left(4\lambda^2+8\lambda+55\right)\left(16\lambda^4+64\lambda^3+24\lambda^2-80\lambda+1225\right)\omega \\ -9600\lambda-2560\lambda^3-73000-640\lambda^4-7360\lambda^2 \end{array}\right)$$

is negative for $\omega \in (\frac{5}{12}, \frac{7}{10}]$.[24] Hence $\gamma_{\lambda 2}$ is concave and $\gamma_{\lambda 2}(\omega) > 0$ in $(\frac{5}{12}, \frac{7}{10}]$ since $\gamma_{\lambda 2}(\frac{5}{12}) = 435000 - \frac{3500}{27}\lambda^9 - \frac{500}{27}\lambda^{10} + \frac{166345375}{576}\lambda + \frac{32639875}{1728}\lambda^2 + \frac{12248125}{108}\lambda^3 + \frac{5045125}{108}\lambda^4 + \frac{236875}{54}\lambda^5 + \frac{1083625}{54}\lambda^6 + \frac{138500}{27}\lambda^7 + \frac{20500}{27}\lambda^8 > 435000 - \frac{3500}{27}(\frac{1}{2})^9 - \frac{500}{27}(\frac{1}{2})^{10} = \frac{1002239375}{2304} > 0$ and $\gamma_2(\frac{7}{10}) = 78000 - \frac{932845}{8}\lambda - \frac{76832}{125}\lambda^9 - \frac{10976}{125}\lambda^{10} + \frac{2701325}{8}\lambda^2 + \frac{1445206}{5}\lambda^3 + \frac{5525534}{25}\lambda^4 + \frac{5330276}{125}\lambda^5 + \frac{6376076}{125}\lambda^6 + \frac{1440992}{125}\lambda^7 + \frac{130144}{125}\lambda^8 > 78000 - \frac{932845}{8}(\frac{1}{2}) - \frac{76832}{125}(\frac{1}{2})^9 - \frac{10976}{125}(\frac{1}{2})^{10} = \frac{15756721}{800} > 0$ for $\lambda \in (0, \frac{1}{2})$.

For $\omega \in (\frac{7}{10}, \frac{5}{6}]$ we have

$$\Gamma[\xi(\omega,\lambda)] = \frac{\omega(1-2\lambda)}{31640625}\left(\begin{array}{c} 2\lambda(1-\lambda)\left(27+17\lambda+8\lambda^2\right)\left(8\lambda^2+17\lambda+102\right)\left(64\lambda^4+272\lambda^3+121\lambda^2-357\lambda+4329\right)\omega^3 \\ -450\lambda(1-\lambda)\left(27+17\lambda+8\lambda^2\right)\left(64\lambda^4+272\lambda^3+721\lambda^2+918\lambda+6354\right)\omega^2 \\ -16875\left(27+17\lambda+8\lambda^2\right)\left(16\lambda^3+26\lambda^2-105\lambda+69\right)\omega \\ -6750000\lambda^2+5906250\lambda+30375000 \end{array}\right)$$

We use $\gamma_{\lambda 3}(\omega)$ to denote the term in parenthesis and notice that

$$\gamma''_{\lambda 3}(\omega) = 12\lambda(1-\lambda)\left(27+17\lambda+8\lambda^2\right)\left(\begin{array}{c} \left(8\lambda^2+17\lambda+102\right)\left(64\lambda^4+272\lambda^3+121\lambda^2-357\lambda+4329\right)\omega \\ -20400\lambda^3-4800\lambda^4-476550-68850\lambda-54075\lambda^2 \end{array}\right)$$

is negative for $\omega \in (\frac{7}{10}, \frac{5}{6}]$.[25] Hence $\gamma_{\lambda 3}$ is concave and $\gamma_{\lambda 3}(\omega) > 0$ for any $\omega \in (\frac{7}{10}, \frac{5}{6}]$ since $\gamma_{\lambda 3}(\frac{7}{10}) = \frac{16736625}{2} - \frac{1027844181}{250}\lambda - \frac{526848}{25}\lambda^9 - \frac{351232}{125}\lambda^{10} + \frac{2863618029}{500}\lambda^2 + \frac{3858009939}{500}\lambda^3 + \frac{244816677}{50}\lambda^4 + \frac{669985309}{500}\lambda^5 + \frac{769607769}{500}\lambda^6 + \frac{46400648}{125}\lambda^7 + \frac{2839648}{125}\lambda^8 \geq \frac{16736625}{2} - \frac{1027844181}{250}(\frac{1}{2}) - \frac{526848}{25}(\frac{1}{2})^9 - \frac{351232}{125}(\frac{1}{2})^{10} = \frac{3156290117}{500} > 0$ and $\gamma_{\lambda 3}(\frac{5}{6}) = \frac{8353125}{2} - \frac{21070125}{2}\lambda + \frac{35516125}{4}\lambda^2 - \frac{320000}{9}\lambda^9 - \frac{128000}{27}\lambda^{10} + \frac{41704875}{4}\lambda^3 + \frac{15135625}{2}\lambda^4 + \frac{229970375}{108}\lambda^5 + \frac{74512625}{36}\lambda^6 + \frac{12487000}{27}\lambda^7 + \frac{212000}{27}\lambda^8 > \frac{8353125}{2} - \frac{21070125}{2}\lambda + \frac{35516125}{4}\lambda^2 - \frac{320000}{9}(\frac{1}{2})^9 - \frac{128000}{27}(\frac{1}{2})^{10}$, which is decreasing in λ and at $\lambda = \frac{1}{2}$ has value $\frac{8353125}{2} - \frac{21070125}{2}(\frac{1}{2}) + \frac{35516125}{4}(\frac{1}{2})^2 - \frac{320000}{9}(\frac{1}{2})^9 - \frac{128000}{27}(\frac{1}{2})^{10} = \frac{487604875}{432} > 0$.

For $\omega \in (\frac{5}{6}, 1]$ we have

$$\Gamma[\xi(\omega,\lambda)] = \frac{\omega(1-2\lambda)}{1280000}\left(\begin{array}{c} 9\lambda(1-\lambda)\left(2\lambda^2+3\lambda+18\right)\left(12\lambda^4+36\lambda^3+3\lambda^2-36\lambda+412\right)\left(14+9\lambda+6\lambda^2\right)\omega^3 \\ -120\lambda(1-\lambda)\left(36\lambda^4+108\lambda^3+249\lambda^2+252\lambda+1796\right)\left(14+9\lambda+6\lambda^2\right)\omega^2 \\ -2400\left(14+9\lambda+6\lambda^2\right)\left(12\lambda^3+12\lambda^2-55\lambda+38\right)\omega+576000\lambda+1408000-384000\lambda^2 \end{array}\right)$$

We use $\gamma_{\lambda 4}(\omega)$ to denote the term in parenthesis and notice that

$$\gamma''_{\lambda 4}(\omega) = 6\lambda(1-\lambda)\left(14+9\lambda+6\lambda^2\right)\left(\begin{array}{c} 9\left(2\lambda^2+3\lambda+18\right)\left(12\lambda^4+36\lambda^3+3\lambda^2-36\lambda+412\right)\omega \\ -4320\lambda^3-1440\lambda^4-10080\lambda-9960\lambda^2-71840 \end{array}\right)$$

[24] γ''_2 is increasing and $\gamma''_2(\frac{7}{10}) = \frac{3}{5}\lambda(1-\lambda)(4\lambda^2+8\lambda+15)(448\lambda^6+2688\lambda^5+4016\lambda^4-1856\lambda^3-34540\lambda^2-58200\lambda-258375) < 0$.

[25] $\gamma''_{\lambda 3}$ is increasing and $\gamma''_{\lambda 3}(\frac{5}{6}) = 10\lambda(1-\lambda)(27+17\lambda+8\lambda^2)(512\lambda^6+3264\lambda^5+6360\lambda^4+2465\lambda^3-23985\lambda^2-45441\lambda-130302) < 0$.

is negative for $\omega \in (\frac{5}{6}, 1]$.[26] Hence $\gamma_{\lambda 4}$ is concave and $\gamma_{\lambda 4}(\omega) > 0$ for any $\omega \in (\frac{5}{6}, 1]$ since $\gamma_{\lambda 4}(\frac{5}{6}) = 344000 - \frac{367750}{3}\lambda - 3750\lambda^9 - 750\lambda^{10} + \frac{354250}{3}\lambda^2 + \frac{889375}{3}\lambda^3 + \frac{1081250}{3}\lambda^4 + \frac{205375}{8}\lambda^5 + \frac{1060625}{8}\lambda^6 + 38500\lambda^7 + 5375\lambda^8 > 344000 - \frac{367750}{3}(\frac{1}{2}) - 3750(\frac{1}{2})^9 - 750(\frac{1}{2})^{10} = \frac{434227625}{1536} > 0$ and $\gamma_{\lambda 4}(1) = 131200 - 479664\lambda + 248208\lambda^2 + 379368\lambda^3 + 542520\lambda^4 + 65313\lambda^5 + 181143\lambda^6 + 48384\lambda^7 + 4104\lambda^8 - 6480\lambda^9 - 1296\lambda^{10} > 0$ for $\lambda \in (0, \frac{1}{2})$. Precisely, the inequality $\gamma_{\lambda 4}(1) > 0$ is not straightforward to prove. It is obvious that $131200 - 6480\lambda^9 - 1296\lambda^{10} \geq 131200 - 6480(\frac{1}{2})^9 - 1296(\frac{1}{2})^{10} = \frac{8395909}{64}$, and the inequality $\frac{8395909}{64} - 479664\lambda + 248208\lambda^2 < 0$ requires $\lambda > \frac{13}{40}$. For $\lambda > \frac{13}{40}$, we have that $\gamma_{\lambda 4}(1) > \frac{8395909}{64} - 479664\lambda + 248208\lambda^2 + 379368(\frac{13}{40})^3 + 542520(\frac{13}{40})^4 + 65313(\frac{13}{40})^5 + 181143(\frac{13}{40})^6 + 48384(\frac{13}{40})^7 + 4104(\frac{13}{40})^8$, and this expression negative requires $\lambda > \frac{7}{18}$. For $\lambda > \frac{7}{18}$ we have that $\gamma_{\lambda 4}(1) > \frac{8395909}{64} - 479664\lambda + 248208\lambda^2 + 379368(\frac{7}{18})^3 + 542520(\frac{7}{18})^4 + 65313(\frac{7}{18})^5 + 181143(\frac{7}{18})^6 + 48384(\frac{7}{18})^7 + 4104(\frac{7}{18})^8$, and this expression negative requires $\lambda > \frac{9}{20}$. For $\lambda > \frac{9}{20}$ we have that $\gamma_{\lambda 4}(1) > \frac{8395909}{64} - 479664(\frac{1}{2}) + 248208(\frac{9}{20})^2 + 379368(\frac{9}{20})^3 + 542520(\frac{9}{20})^4 + 65313(\frac{9}{20})^5 + 181143(\frac{9}{20})^6 = \frac{7\,31005\,07803}{64000000} > 0$.

Step 7 If $\xi(\omega, \lambda)$ is such that $\Psi[\xi(\omega, \lambda)] < 0$ and $\Gamma[\xi(\omega, \lambda)] < 0$ for any $\omega \in (0, 1]$ and any $\lambda \in (\frac{1}{2}, 1)$, then $U_{PR}^B > U_C^B$ for any $\omega \in (0, 1]$ and any $\lambda \in (\frac{1}{2}, 1)$.
Proof From $\Psi[\xi(\omega, \lambda)] < 0$ and Step 1 we infer that $\xi(\omega, \lambda) < \hat{x}$. From $\Gamma[\xi(\omega, \lambda)] < 0$ and Step 3 we infer that $\Gamma(\hat{x}) < 0$, thus $U_{PR}^B > U_C^B$ by Step 2 since $\lambda > \frac{1}{2}$.

Step 8 Let $\xi(\omega, \lambda) \equiv \begin{cases} \frac{3}{8}\omega + \frac{1}{4}\omega\lambda & \text{for } \omega \in (0, \frac{58}{100}] \\ \frac{1}{3}\omega + \frac{1}{3}\omega\lambda & \text{for } \omega \in (\frac{58}{100}, \frac{7}{8}] \\ \frac{4}{15}\omega + \frac{8}{15}\omega\lambda - \frac{2}{15}\omega\lambda^2 & \text{for } \omega \in (\frac{7}{8}, 1] \end{cases}$. Then $\Psi[\xi(\omega, \lambda)] < 0$ for any $\omega \in (0, 1]$ and any $\lambda \in (\frac{1}{2}, 1]$.

Proof For $\omega \in (0, \frac{58}{100}]$ we have

$$\Psi[\xi(\omega, \lambda)] = \frac{\omega\,(2\lambda - 1)}{128}\left(\lambda\,(1 - \lambda)\,(3 + 2\lambda)^2\,\omega^2 + 2\,(3 + 2\lambda)\left(4\lambda^2 + 4\lambda - 7\right)\omega + 16 - 32\lambda\right)$$

We use $\psi_{\lambda 5}(\omega)$ to denote the term inside the parenthesis and notice that $\psi_{\lambda 5}(\omega) < 0$ for any $\omega \in (0, \frac{58}{100}]$ since $\psi_{\lambda 5}$ is convex and $\psi_{\lambda 5}(0) = 16 - 32\lambda < 0$, $\psi_{\lambda 5}(\frac{58}{100}) = -\frac{209}{25} - \frac{78231}{2500}\lambda + \frac{60523}{2500}\lambda^2 + \frac{4118}{625}\lambda^3 - \frac{841}{625}\lambda^4 < 0$ for $\lambda \in (\frac{1}{2}, 1)$.[27]

For $\omega \in (\frac{58}{100}, \frac{7}{8})$ we have

$$\Psi[\xi(\omega, \lambda)] = \frac{(2\lambda - 1)\,(1 - \lambda)\,\omega}{27}\left(2\lambda\,(\lambda + 1)^2\,\omega^2 - 3\,(2\lambda + 3)\,(\lambda + 1)\,\omega + 9\right)$$

We use $\psi_{\lambda 6}(\omega)$ to denote the term inside the parenthesis and notice that $\psi_{\lambda 6}(\omega) < 0$ for any $\omega \in (\frac{58}{100}, \frac{7}{8}]$ since $\psi_{\lambda 6}$ is convex and $\psi_{\lambda 6}(\frac{58}{100}) = \frac{189}{50} - \frac{5017}{625}\lambda - \frac{1334}{625}\lambda^2 + \frac{841}{1250}\lambda^3 < \frac{189}{50} - \frac{5017}{625}(\frac{1}{2}) - \frac{1334}{625}(\frac{1}{2})^2 + \frac{841}{1250} = -\frac{59}{625} < 0$, $\psi_{\lambda 6}(\frac{7}{8}) = \frac{49}{32}\lambda^3 - \frac{35}{16}\lambda^2 - \frac{371}{32}\lambda + \frac{9}{8} < \frac{49}{32} - \frac{35}{16}(\frac{1}{2})^2 - \frac{371}{32}(\frac{1}{2}) + \frac{9}{8} = -\frac{59}{16} < 0$. for $\lambda \in (\frac{1}{2}, 1)$.

For $\omega \in (\frac{7}{8}, 1)$ we have

$$\Psi[\xi(\omega, \lambda)] = \frac{2\omega\,(2\lambda - 1)\,(1 - \lambda)}{3375}\left(4\lambda\,(7 - 2\lambda)\left(\lambda^2 - 4\lambda - 2\right)^2\omega^2 - 30\,(9 - 2\lambda)\,(\lambda + 1)\left(2 - \lambda^2 + 4\lambda\right)\omega - 225\lambda + 1125\right)$$

Let $\psi_{\lambda 7}(\omega)$ denote the term inside the parenthesis and notice that $\psi_{\lambda 7}(\omega) < 0$ for any $\omega \in (\frac{7}{8}, 1)$ since $\psi_{\lambda 7}$ is convex and $\psi_{\lambda 7}(\frac{7}{8}) = \frac{1305}{2} - \frac{5807}{4}\lambda - \frac{301}{4}\lambda^2 + 553\lambda^3 - \frac{595}{2}\lambda^4 + \frac{1127}{16}\lambda^5 - \frac{49}{8}\lambda^6 < 0$, $\psi_{\lambda 7}(1) = 585 - 1613\lambda - 34\lambda^2 + 658\lambda^3 - 380\lambda^4 + 92\lambda^5 - 8\lambda^6 < 0$ for $\lambda \in (\frac{1}{2}, 1)$.

[26] $\gamma''_{\lambda 4}$ is increasing and $\gamma''_{\lambda 4}(1) = -6\lambda(1 - \lambda)(26 - 6\lambda^2 - 9\lambda)(14 + 9\lambda + 6\lambda^2)^3 < 0$.
[27] Precisely, $\psi_{\lambda 5}(\frac{58}{100})$ is convex in λ and has negative value at $\lambda = \frac{1}{2}$ $(-\frac{10759}{625})$ and at $\lambda = 1$ $(-\frac{51}{5})$.

Step 9 For the same $\xi(\omega,\lambda)$ introduced in Step 8, the inequality $\Gamma[\xi(\omega,\lambda)]<0$ holds for any $\omega\in(0,1]$ and any $\lambda\in(\frac{1}{2},1]$. In view of Steps 7 and 8, we conclude that $U_{PR}^B>U_C^B$ for any $\omega\in(0,1]$ and any $\lambda\in(\frac{1}{2},1)$.
Proof For $\omega\in(0,\frac{58}{100}]$ we have

$$\Gamma[\xi(\omega,\lambda)]=\frac{\omega(2\lambda-1)}{2048}\begin{pmatrix}-\lambda(2\lambda+11)(1-\lambda)(2\lambda+3)\left(4\lambda^2-4\lambda+49\right)\omega^3\\+24\lambda(1-\lambda)(2\lambda+3)\left(4\lambda^2+12\lambda+73\right)\omega^2\\+96(2\lambda+3)\left(4\lambda^2-12\lambda+7\right)\omega+1024\lambda-1536\end{pmatrix}$$

We use $\gamma_{\lambda5}(\omega)$ to denote the term in parenthesis and notice that $\gamma_{\lambda5}''(\omega)=-6\lambda(2\lambda+11)(1-\lambda)(2\lambda+3)(4\lambda^2-4\lambda+49)\omega+48\lambda(1-\lambda)(2\lambda+3)\left(4\lambda^2+12\lambda+73\right)$ is positive for $\omega\in(0,\frac{58}{100}]$.[28] Hence $\gamma_{\lambda5}$ is convex and $\gamma_{\lambda5}(\omega)<0$ for any $\omega\in(0,\frac{58}{100}]$ because $\gamma_{\lambda5}(0)=1024\lambda-1536<0$ and $\gamma_{\lambda5}(\frac{58}{100})=-\frac{9168}{25}+\frac{156457787}{125000}\lambda-\frac{111665747}{125000}\lambda^2-\frac{8336108}{15625}\lambda^3-\frac{633273}{3125}\lambda^4-\frac{153062}{3125}\lambda^5+\frac{48778}{15625}\lambda^6<-\frac{9168}{25}+\frac{156457787}{125000}\lambda-\frac{111665747}{125000}\lambda^2-\frac{8336108}{15625}(\frac{1}{2})^3-\frac{633273}{3125}(\frac{1}{2})^4-\frac{153062}{3125}(\frac{1}{2})^5+\frac{48778}{15625}=\frac{1}{62500}(-27780197+\frac{156457787}{2}\lambda-\frac{111665747}{2}\lambda^2)<0$ for any $\lambda\in(\frac{1}{2},1)$.

For $\omega\in(\frac{58}{100},\frac{7}{8}]$ we have

$$\Gamma[\xi(\omega,\lambda)]=\frac{1}{81}\omega(2\lambda-1)\begin{pmatrix}-2\lambda(1-\lambda)(\lambda+4)(\lambda+1)\left(\lambda^2-\lambda+7\right)\omega^3+18\lambda(1-\lambda)(\lambda+1)\left(\lambda^2+2\lambda+10\right)\omega^2\\+27(1-\lambda)(3-2\lambda)(\lambda+1)\omega+54\lambda-108\end{pmatrix}$$

We use $\gamma_{\lambda6}(\omega)$ to denote the term in parenthesis and notice that $\gamma_{\lambda6}''(\omega)=-12\lambda(1-\lambda)(\lambda+4)(\lambda+1)(\lambda^2-\lambda+7)\omega+36\lambda(1-\lambda)(\lambda+1)(\lambda^2+2\lambda+10)$ is positive.[29] Hence $\gamma_{\lambda6}$ is convex and $\gamma_{\lambda6}(\omega)<0$ for any $\omega\in(\frac{58}{100},\frac{7}{8}]$ because $\gamma_{\lambda6}(\frac{58}{100})=-\frac{3051}{50}+\frac{1129777}{15625}\lambda-\frac{2252517}{62500}\lambda^2-\frac{33553}{2500}\lambda^3-\frac{354061}{31250}\lambda^4-\frac{305283}{62500}\lambda^5+\frac{24389}{62500}\lambda^6<-\frac{3051}{50}+\frac{1129777}{15625}\lambda-\frac{2252517}{62500}\lambda^2-\frac{33553}{2500}(\frac{1}{2})^3-\frac{354061}{31250}(\frac{1}{2})^4-\frac{305283}{62500}(\frac{1}{2})^5+\frac{24389}{62500}=-\frac{126336379}{2000000}+\frac{1129777}{15625}\lambda-\frac{2252517}{62500}\lambda^2<0$ and $\gamma_{\lambda6}(\frac{7}{8})=-\frac{2499}{256}\lambda^5-\frac{3185}{128}\lambda^4-\frac{11081}{256}\lambda^3-\frac{12117}{256}\lambda^2+\frac{6851}{64}\lambda+\frac{343}{256}\lambda^6-\frac{297}{8}<0$ for any $\lambda\in(\frac{1}{2},1)$.

For $\omega\in(\frac{7}{8},1]$ we have

$$\Gamma[\xi(\omega,\lambda)]=\frac{2\omega(2\lambda-1)}{50625}\begin{pmatrix}-2\lambda(1-\lambda)\left(4\lambda+2-\lambda^2\right)\left(8\lambda+19-2\lambda^2\right)\left(4\lambda^4-32\lambda^3+78\lambda^2-56\lambda+181\right)\omega^3\\+90\lambda(\lambda-1)\left(\lambda^2-4\lambda-2\right)\left(4\lambda^4-32\lambda^3+48\lambda^2+64\lambda+241\right)\omega^2\\+1350(\lambda-1)\left(\lambda^2-4\lambda-2\right)\left(2\lambda^2-7\lambda+9\right)\omega-6750\lambda^2+47250\lambda-57375\end{pmatrix}$$

We use $\gamma_{\lambda7}(\omega)$ to denote the term inside the parenthesis and notice that

$$\begin{aligned}\gamma_{\lambda7}''(\omega)&=-12\lambda(1-\lambda)\left(4\lambda+2-\lambda^2\right)\left(8\lambda+19-2\lambda^2\right)\left(4\lambda^4-32\lambda^3+78\lambda^2-56\lambda+181\right)\omega\\&\quad+180\lambda(\lambda-1)\left(\lambda^2-4\lambda-2\right)\left(4\lambda^4-32\lambda^3+48\lambda^2+64\lambda+241\right)\end{aligned}$$

is positive.[30] Hence $\gamma_{\lambda7}$ is convex and $\gamma_{\lambda7}(\omega)<0$ for any $\omega\in(\frac{7}{8},1]$ because $\gamma_{\lambda7}(\frac{7}{8})=-\frac{72225}{2}+\frac{9724463}{128}\lambda-\frac{5111089}{128}\lambda^2+\frac{2383829}{256}\lambda^3-\frac{7039697}{256}\lambda^4-9079\lambda^5+15484\lambda^6-\frac{49931}{8}\lambda^7+\frac{11123}{8}\lambda^8-\frac{5831}{32}\lambda^9+\frac{343}{32}\lambda^{10}<0$, and $\gamma_{\lambda7}(1)=-33075+82274\lambda-41392\lambda^2+6676\lambda^3-33418\lambda^4-11396\lambda^5+20336\lambda^6-8648\lambda^7+2024\lambda^8-272\lambda^9+16\lambda^{10}<0$ for any $\lambda\in(\frac{1}{2},1)$.

6.7 Proof of Proposition 5

(i) From the remarks stated shortly before Proposition 5 we see that (i) for $(q_1,q_2)\in Q_3$, $u_{PU}^B(q_1,q_2)-u_{PR}^B(q_1,q_2)=(1-\lambda)\sigma$; (ii) for $(q_1,q_2)\in Q_1\cup Q_2$, $u_{PU}^B(q_1,q_2)-u_{PR}^B(q_1,q_2)>-\lambda\sigma$ since $u_{PU}^B(q_1,q_2)>$

[28]Precisely, $\gamma_{\lambda5}''$ is decreasing and $\gamma_{\lambda5}''(\frac{58}{100})=\frac{3}{25}\lambda(1-\lambda)(2\lambda+3)(13569+3234\lambda+556\lambda^2-232\lambda^3)>0$.
[29]Precisely, $\gamma_{\lambda6}''$ is decreasing and $\gamma_{\lambda6}''(\frac{7}{8})=\frac{3}{2}\lambda(1-\lambda)(\lambda+1)(44+27\lambda+3\lambda^2-7\lambda^3)>0$.
[30]Precisely, $\gamma_{\lambda7}''$ is decreasing and $\gamma_{\lambda7}''(1)=48\lambda(1-\lambda)(2\lambda^2-8\lambda+11)(-\lambda^2+4\lambda+2)^3>0$.

$q_1 - c_H$ and $u^B_{PR}(q_1, q_2) \leq q_1 - c_L - (1-\lambda)\sigma$. Therefore

$$U^B_{PU} - U^B_{PR} > 2[(1-\lambda)\sigma \Pr\{(q_1,q_2) \in Q_3\} - \lambda\sigma \Pr\{(q_1,q_2) \in Q_1 \cup Q_2\}] = 2\sigma[\Pr\{(q_1,q_2) \in Q_3\} - \frac{\lambda}{2}]$$

in which the equality holds since $\Pr\{(q_1,q_2) \in Q_1 \cup Q_2 \cup Q_3\} = \frac{1}{2}$ and thus $\Pr\{(q_1,q_2) \in Q_1 \cup Q_2\} = \frac{1}{2} - \Pr\{(q_1,q_2) \in Q_3\}$. For σ about 0 we find that $\Pr\{(q_1,q_2) \in Q_3\}$ is about $\frac{1}{2}$, and hence $U^B_{PU} - U^B_{PR} > 0$.
(ii) We prove that $U^B_{PR} > U^B_{PU}$ when $\omega > \max\{\frac{3(1+\lambda)}{2(3-\lambda)}, \frac{3\lambda-1}{2(1-\lambda)}\}$ by showing that $u^B_{PR}(q_1,q_2) > u^B_{PU}(q_1,q_2)$ for any feasible (q_1,q_2). Precisely, we show that for any feasible (q_1,q_2), $u^B_{PR}(q_1,q_2)$ is larger than B's payoff under PU even if we suppose that the highest valuation supplier always wins under PU.[31] That is equivalent to prove that the suppliers' rents are larger in PU than in PR. Notice that $\max\{\frac{3(1+\lambda)}{2(3-\lambda)}, \frac{3\lambda-1}{2(1-\lambda)}\} = \begin{cases} \frac{3(1+\lambda)}{2(3-\lambda)} & \text{if } \lambda \leq \frac{3}{5} \\ \frac{3\lambda-1}{2(1-\lambda)} & \text{if } \lambda > \frac{3}{5} \end{cases}$.

Step 1 The case of $\lambda \leq \frac{3}{5}$ Suppose that $\omega \geq \frac{3(1+\lambda)}{2(3-\lambda)}$ and notice that $\frac{3(1+\lambda)}{2(3-\lambda)} \leq 1$ when $\lambda \leq \frac{3}{5}$.

Step 1.1 $\omega \geq 1$ The condition $\omega \geq 1$ is equivalent to $\sigma \geq \theta$, thus any feasible (q_1,q_2) (such that $q_2 \geq q_1$) belongs to Q_1 and from Corollary 2 we infer that

$$\begin{aligned} u^B_{PU}(q_1,q_2) &= q_1 - c_L + \lambda(2-\lambda)t - (1-\lambda^2)\sigma - \lambda\sqrt{[t+(1-\lambda)\sigma]^2 - 4\lambda(1-\lambda)t^2} \\ &\quad -\lambda^2 t \Pr\{1_L \text{ def } 2_L\} + \lambda(1-\lambda)(\sigma - t)\Pr\{1_L \text{ def } 2_H\} \end{aligned} \quad (24)$$

Since $\sigma - t \geq \sigma - \theta \geq 0$ and $\Pr\{1_L \text{ def } 2_H\} < 1$, if follows that $\lambda(1-\lambda)(\sigma-t)\Pr\{1_L \text{ def } 2_H\} \leq \lambda(1-\lambda)(\sigma-t)$. Using $\lambda^2 t \Pr\{1_L \text{ def } 2_L\} > 0$ we conclude that $u^B_{PU}(q_1,q_2) < q_1 - c_L + \lambda(2-\lambda)t - (1-\lambda^2)\sigma - \lambda\sqrt{[t+(1-\lambda)\sigma]^2 - 4\lambda(1-\lambda)t^2} + \lambda(1-\lambda)(\sigma-t) = q_1 - c_L - (1-\lambda)\sigma + \lambda t - \lambda\sqrt{[t+(1-\lambda)\sigma]^2 - 4\lambda(1-\lambda)t^2}$. Moreover, for each $(q_1,q_2) \in Q_1$ we know that $u^B_{PR}(q_1,q_2) = q_1 - c_L - (1-\lambda)\sigma - \lambda(1-\lambda)(\sigma-t)$ and thus $u^B_{PU}(q_1,q_2) < u^B_{PR}(q_1,q_2)$ holds if $q_1 - c_L - (1-\lambda)\sigma + \lambda t - \lambda\sqrt{[t+(1-\lambda)\sigma]^2 - 4\lambda(1-\lambda)t^2} \leq q_1 - c_L - (1-\lambda)\sigma - \lambda(1-\lambda)(\sigma-t)$. This inequality is equivalent to $2(1-\lambda)\sigma \geq (3\lambda-1)t$, which is satisfied for any $t \in [0,\theta]$, given $\sigma \geq \theta$ and $\lambda \leq \frac{3}{5}$.

Step 1.2 $\frac{3(1+\lambda)}{2(3-\lambda)} \leq \omega < 1$ The inequality $\omega \geq \frac{3(1+\lambda)}{2(3-\lambda)}$ is equivalent to $\sigma \geq \frac{3(1+\lambda)}{2(3-\lambda)}\theta$, and since $\lambda < \frac{3(1+\lambda)}{2(3-\lambda)}$, it implies that $\sigma > \lambda\theta$; thus $q_2 - q_1 \leq \frac{\sigma}{\lambda}$ holds for any feasible (q_1,q_2). Hence any feasible (q_1,q_2) (such that $q_2 \geq q_1$) belongs to Q_1 if $q_2 - q_1 \leq \sigma$, otherwise it belongs to Q_2.
If $(q_1,q_2) \in Q_1$, using $\sigma \geq t = q_2 - q_1$ we can argue like in Step 1.1 to find that the inequality $2(1-\lambda)\sigma \geq (3\lambda-1)t$ implies $u^B_{PU}(q_1,q_2) < u^B_{PR}(q_1,q_2)$. The inequality $2(1-\lambda)\sigma \geq (3\lambda-1)t$ holds for any $(q_1,q_2) \in Q_1$ given that $\sigma \geq t$ and $\lambda \leq \frac{3}{5}$.
If conversely $(q_1,q_2) \in Q_2$, then $\sigma - t < 0$ and therefore $u^B_{PU}(q_1,q_2) < q_1 - c_L + \lambda(2-\lambda)t - (1-\lambda^2)\sigma - \lambda\sqrt{[t+(1-\lambda)\sigma]^2 - 4\lambda(1-\lambda)t^2}$. Moreover, for each $(q_1,q_2) \in Q_2$ we know that $u^B_{PR}(q_1,q_2) = q_1 - c_L - (1-\lambda)\sigma$ and thus $u^B_{PU}(q_1,q_2) < u^B_{PR}(q_1,q_2)$ holds if $q_1 - c_L + \lambda(2-\lambda)t - (1-\lambda^2)\sigma - \lambda\sqrt{[t+(1-\lambda)\sigma]^2 - 4\lambda(1-\lambda)t^2} \leq q_1 - c_L - (1-\lambda)\sigma$. This inequality is equivalent to $3(1+\lambda)t \leq 2(3-\lambda)\sigma$, which is satisfied for any $t \in [\sigma,\theta]$ given $\sigma \geq \frac{3(1+\lambda)}{2(3-\lambda)}\theta$.

Step 2 The case of $\lambda > \frac{3}{5}$ Assume that $\omega \geq \frac{3\lambda-1}{2(1-\lambda)}$ and notice that $\frac{3\lambda-1}{2(1-\lambda)} > 1$ when $\lambda > \frac{3}{5}$. Then $\omega > 1$ holds and we can argue as in Step 1.1 to prove that the inequality $2(1-\lambda)\sigma \geq (3\lambda-1)t$ implies $u^B_{PU}(q_1,q_2) < u^B_{PR}(q_1,q_2)$. The inequality $2(1-\lambda)\sigma \geq (3\lambda-1)t$ holds for any $t \in [0,\theta]$ given $\omega \geq \frac{3\lambda-1}{2(1-\lambda)}$.

[31] That is, even if $\Pr\{1_L \text{ def } 2_L\} = 0$ and $\Pr\{1_L \text{ def } 2_H\} = 1$ for $q_1 - c_L > q_2 - c_H$, $\Pr\{1_L \text{ def } 2_H\} = 0$ for $q_1 - c_L < q_2 - c_H$.

6.8 Proof of Proposition 6(ii)

Given $\lambda = \frac{1}{2}$, in this proof we show that $U^B_{PU} > U^B_{PR}$ if $\omega < \omega^*$ and $U^B_{PU} < U^B_{PR}$ if $\omega > \omega^*$. Then Proposition 4 implies straightforwardly Proposition 6.

From Proposition 5(ii), with $\lambda = \frac{1}{2}$, we know that $U^B_{PR} > U_{PU}$ for $\omega \geq \frac{9}{10}$. Hence, for $\omega < \frac{9}{10}$ we compare U^B_{PU} with $U^B_{PR} = \underline{q} - c_L + \frac{1}{12}(4 - 6\omega - 3\omega^2 + \omega^3)\theta$ [derived in Corollary 1(i)]. Recall from Subsection 3.3 that t denotes the difference $q_2 - q_1$, and to fix the ideas (without loss of generality) we assume that $t \geq 0$. Given that $\lambda = \frac{1}{2}$, the inequality $\lambda t < \sigma$ reduces to $t < 2\sigma$ and thus the set Q_3 is empty if $\omega > \frac{1}{2}$, whereas $Q_3 \neq \emptyset$ if $\omega \leq \frac{1}{2}$. Therefore we distinguish these two cases.

Step 1 The case of $\omega \leq \frac{1}{2}$ We need to derive U^B_{PU}. From $\omega \leq \frac{1}{2}$ we deduce that Q_1, Q_2, Q_3 are all non-empty sets, and when $(q_1, q_2) \in Q_1 \cup Q_2$ – that is when $t < 2\sigma$ – $u^B_{PU}(q_1, q_2)$ is given by (24) with $\lambda = \frac{1}{2}$. However, we can simplify this expression somewhat by defining $\tau \equiv \frac{t}{\sigma}$, that is $\tau = \frac{q_2 - q_1}{\sigma}$, since then from (21) and $\hat{p} = (1-\lambda)\tilde{p} + \lambda(t + c_L)$ we obtain $\tilde{p} = c_L + \frac{1}{2}(1 + \sqrt{1+4\tau})\sigma$, $\hat{p} = c_L + \frac{1}{4}(2\tau + 1 + \sqrt{1+4\tau})\sigma$, and using (**??**), (**??**) we find

$$\Pr\{1_L \text{ def } 2_L\} = \frac{1}{2\tau^2}(1 + 2\tau - 2\tau^2 + \sqrt{1+4\tau}) \ln \frac{2\tau + 1 + \sqrt{1+4\tau}}{1 + \sqrt{1+4\tau}} - \frac{1 + \sqrt{1+4\tau} - 2\tau}{2\tau}$$

$$\Pr\{1_L \text{ def } 2_H\} = \frac{\sqrt{1+4\tau} - 3}{2(\tau - 1)} + \tau \frac{1 - 2\tau + \sqrt{1+4\tau}}{2(\tau-1)^2} \ln \frac{\sqrt{1+4\tau} - 1}{\tau(1 + \sqrt{1+4\tau} - 2\tau)}$$

Therefore $u^B_{PU}(q_1, q_2) = q_1 - c_L + \frac{\omega}{4}\theta\eta(\tau)$ with

$$\eta(\tau) \equiv \tau \frac{1 + \sqrt{1+4\tau} - 2\tau}{2(1-\tau)} \ln \frac{\sqrt{1+4\tau} - 1}{\tau(1 + \sqrt{1+4\tau} - 2\tau)} - \frac{1 + \sqrt{1+4\tau} + 2\tau - 2\tau^2}{2\tau} \ln \frac{2\tau + 1 + \sqrt{1+4\tau}}{1 + \sqrt{1+4\tau}} + 2\tau - 1 - \sqrt{1+4\tau}$$

We have thus obtained $u^B_{PU}(q_1, q_2)$ when $\tau < 2$. When instead $\tau \geq 2$, Corollary 2 reveals that $u^B_{PU}(q_1, q_2) = q_1 - c_L$; hence

$$u^B_{PU}(q_1, q_2) = \begin{cases} q_1 - c_L & \text{if } \tau \geq 2 \\ q_1 - c_L + \frac{\omega}{4}\theta\eta(\tau) & \text{if } \tau < 2 \end{cases}$$

Ex ante, from the point of view of the buyer, τ is a random variable with support $[0, \frac{1}{\omega}]$, given our assumption of $t \geq 0$, and using the fact that the density for (q_1, q_2) is constantly equal to $\frac{2}{\theta^2}$ in $Q_1 \cup Q_2 \cup Q_3$, we can obtain the c.d.f. Υ and the density Υ' of τ, for $\tau \in [0, \frac{1}{\omega}]$:

$$\Upsilon(\tau) = 2\omega\tau - \omega^2\tau^2 \qquad \Upsilon'(\tau) = 2\omega - 2\omega^2\tau$$

Thus $U^B_{PU} = \int_{\underline{q}}^{\underline{q}+\theta} \int_{q_1}^{\underline{q}+\theta} (q_1 - c_L)\frac{2}{\theta^2} dq_2 dq_1 + \int_0^2 \frac{\omega}{4}\theta\eta(\tau)(2\omega - 2\omega^2\tau)d\tau = \underline{q} - c_L + (\frac{1}{3} + \int_0^2 [\frac{1}{2}\eta(\tau)\omega^2 - \frac{1}{2}\eta(\tau)\tau\omega^3]d\tau)\theta$, and using numerical methods we find $\int_0^2 \frac{1}{2}\eta(\tau)d\tau = -2.15462$, $\int_0^2 (-\frac{1}{2})\eta(\tau)\tau d\tau = 1.70024$; hence $U^B_{PU} = \underline{q} - c_L + (\frac{1}{3} - 2.15462\omega^2 + 1.70024\omega^3)\theta$. Since $U^B_{PR} = \underline{q} - c_L + \frac{1}{12}(4 - 6\omega - 3\omega^2 + \omega^3)\theta$, numerical methods reveal that $U^B_{PU} > U^B_{PR}$ for $\omega < \omega^*$, and $U^B_{PR} > U^B_{PU}$ for $\omega \in (\omega^*, \frac{1}{2}]$.

Step 2 The case of $\omega \in (\frac{1}{2}, \frac{9}{10})$ We prove that $U^B_{PR} > U_{PU}$ for each $\omega \in (\frac{1}{2}, \frac{9}{10})$. Since $\omega > \frac{1}{2}$, the inequality $t < 2\sigma$ holds for any feasible (q_1, q_2) and thus $u^B_{PU}(q_1, q_2)$ is given by (24) with $\lambda = \frac{1}{2}$. Hence $u^B_{PU}(q_1, q_2) < q_1 - c_L + \frac{3}{4}t - \frac{3}{4}\sigma - \frac{1}{2}\sqrt{t\sigma + \frac{1}{4}\sigma^2} + \frac{1}{4}(\sigma - t)$ if $t \leq \sigma$ (as $\Pr\{1_L \text{ def } 2_L\} > 0$ and $\Pr\{1_L \text{ def } 2_H\} < 1$) that is $u^B_{PU}(q_1, q_2) < q_1 - c_L - \frac{1}{2}\omega\theta(1 - \tau + \frac{1}{2}\sqrt{1+4\tau})$ if $\tau \leq 1$; on the other hand, $u^B_{PU}(q_1, q_2) < q_1 - c_L + \frac{3}{4}t - \frac{3}{4}\sigma - \frac{1}{2}\sqrt{t\sigma + \frac{1}{4}\sigma^2}$ if $t > \sigma$ (as $\Pr\{1_L \text{ def } 2_L\} > 0$ and $\Pr\{1_L \text{ def } 2_H\} > 0$)

that is $u_{PU}^B(q_1,q_2) < q_1 - c_L - \frac{1}{2}\omega\theta(\frac{3}{2} - \frac{3}{2}\tau + \frac{1}{2}\sqrt{1+4\tau})$ if $\tau > 1$. Therefore $U_{PU}^B < \int_{\underline{q}}^{\underline{q}+\theta}\int_{q_1}^{\underline{q}+\theta}(q_1 - c_L)\frac{2}{\theta^2}dq_2dq_1 + \int_0^1(-\frac{1}{2}\omega\theta)(1-\tau+\frac{1}{2}\sqrt{1+4\tau})(2\omega-2\omega^2\tau)d\tau + \int_1^{\frac{1}{\omega}}(-\frac{1}{2}\omega\theta)(\frac{3}{2}-\frac{3}{2}\tau+\frac{1}{2}\sqrt{1+4\tau})(2\omega-2\omega^2\tau)d\tau = \underline{q} - c_L + (\frac{7}{12} - \frac{3}{4}\omega + \frac{1}{3}\omega^2 - \frac{3}{40}\omega^3 - \frac{1}{120}(\omega+4)^2\sqrt{\omega(\omega+4)})\theta$. Since $U_{PR}^B = \underline{q} - c_L + \frac{1}{12}(4 - 6\omega - 3\omega^2 + \omega^3)\theta$, the inequality $U_{PR}^B > U_{PU}^B$ reduces to $d_3(\omega) \equiv -3 + 3\omega - 7\omega^2 + \frac{19}{10}\omega^3 + \frac{1}{10}(\omega+4)^2\sqrt{\omega(\omega+4)} > 0$. For each $\omega \in (\frac{1}{2}, \frac{9}{10})$ the inequality $\sqrt{\omega(\omega+4)} > \frac{3}{4} + \frac{3}{2}\omega$ holds, thus $d_3(\omega) > -3 + 3\omega - 7\omega^2 + \frac{19}{10}\omega^3 + \frac{1}{10}(\omega+4)^2(\frac{3}{4} + \frac{3}{2}\omega) = -\frac{9}{5} + 6\omega - \frac{229}{40}\omega^2 + \frac{41}{20}\omega^3 \equiv d_4(\omega)$, and $d_4(\omega) > 0$ for each $\omega \in (\frac{1}{2}, \frac{9}{10})$ since d_4 is increasing and $d_4(\frac{1}{2}) = \frac{1}{40} > 0$. Hence $d_3(\omega) > 0$ for any $\omega \in (\frac{1}{2}, \frac{9}{10})$.

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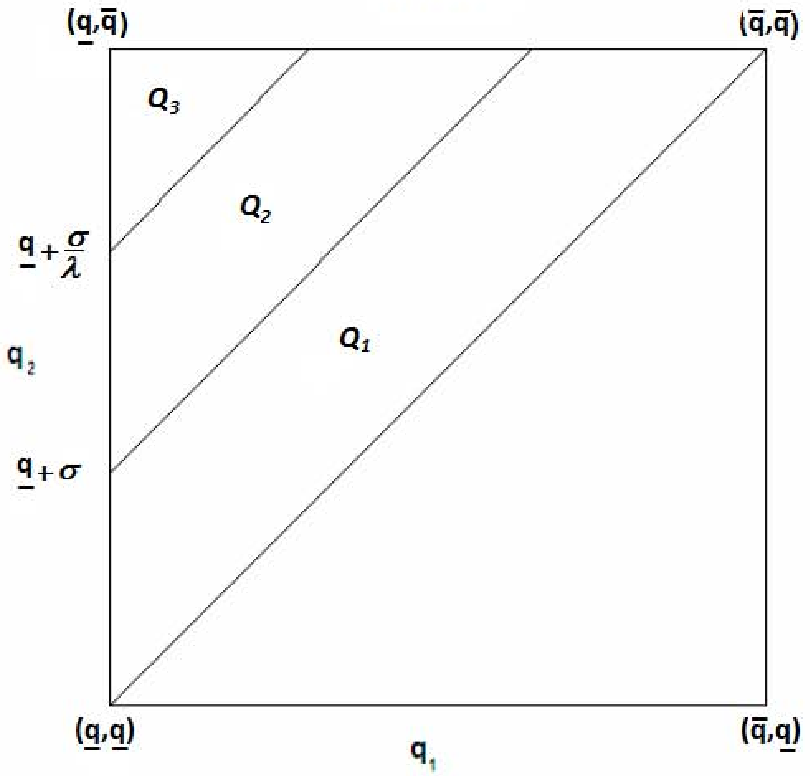
Figure 1



$(\underline{q},\overline{q})$
$(\overline{q},\overline{q})$
Q_3
Q_2
$\underline{q}+\frac{\sigma}{\lambda}$
Q_1
q_2
$\underline{q}+\sigma$
$(\underline{q},\underline{q})$
$(\overline{q},\underline{q})$
q_1