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Nicola Doni

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Dipartimento di Scienze Economiche, Università degli Studi di Firenze
Via delle Pandette 9, 50127 Firenze, Italia
www.dse.unifi.it

A Comparison of Alternative Procedures for the Selection of the Private Partner in PPP Projects

Nicola Doni*
Dipartimento di Scienze Economiche
Università degli Studi di Firenze
nicola.doni@unifi.it

Abstract

In this work we compare three alternative procedures aimed at selecting a private partner in PPP projects: i.e. negotiation, auction and competitive negotiation. We show how the suitability of each of these selection mechanisms depends on many economic and institutional factors: e.g. the extent of contractual complexity, the degree of heterogeneity in firm costs, the level of competition, the probability of corruption. The main lesson of the paper is that the adoption of competitive negotiation can improve public welfare only if the institutional framework can ensure both an actual contestability of each contract and a low risk of corruption phenomena.

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1. Introduction

Public contractors are usually selected by means of public auctions or negotiation. The former kind of procedure is generally reserved for standard contracts and it is characterised by the full objectivity of the awarding criteria. By contrast, the second procedure is mainly adopted in the case of complex contracts, like concessions. Its main peculiarity is that it allows contracting authorities to exert a large discretionary power in deciding which firm to select. Evidently, there is a common view according to which in case of standard procurement, social welfare is better fostered by competitive forces; on the other hand, in case of concessions, it is considered more adequate to rely on the so called "intuitus personae" of public administrators.

However, the recent expansion of public-private partenership (PPP) in many areas of public service provision represents a great challenge from an institutional point of view. In fact, this novelty entails the necessity of more proper rules and procedures. UK has been one of the first country were PPP has been strongly developed and there the Government devised a new kind of selection procedure, known as "competitive negotiation". Recently also EU Commission has introduced a new procedure, called "competitive dialogue" recommended especially for complex contracts and for contractual PPP[1]. In both cases, the aim of these new procedures is to take advantage of the merits associated to price competition without losing flexibility. In fact, as in case of negotiation, this kind of rules allows the contracting authority to choose the contractor not only on the basis of economic bids, but also of its discretionary evaluation.

Why is flexibility so relevant for public procurers? Some empirical research works have examined the possible factors motivating the preference for either negotiation or auction procedures. Bonaccorsi et *al*. (2003) use data on procurement in Italian hospitals to show how the choice between auctions or negotiations is strongly affected by quality-related concerns. Bajari et *al*. (2006) also provide an empirical analysis of the private construction sector in Northern California. One of their main conclusions is that negotiations are generally used to award complex contracts, while auctions are usually reserved for standard works.

It is thus possible to assume that, when a contract is very complex and the quality of the execution is not easily verifiable, contracting authorities want to base

[1] See European Directives 2004/18/EC, art. 29 and COM(2005) 569 final.

their choice not only on the economic proposals, but also on the reliability, of each firm. The problem is that this aspect cannot be measured exclusively on the basis of objective factors. As stated in many existing guidelines[2], the reliability of firms should be assessed not only on the record of their technical capability or on their financial standing, but also on the basis of their past performances, on their references, on their attitude to teamworking and to innovation. These elements inevitably require a subjective evaluation and this is the main motivation for the flexibility entailed in a selection rule. At the same time, it is clear that if we consider the risk of corruption, it is not very prudent to let public officials choose a private contractor only on the grounds of their discretionary evaluation. That is why these new procedures try to combine the flexibility of negotiation with certain characteristics of a standard auction.

In this work we wish to compare three different selection procedures, defined as negotiation, auction and competitive negotiation. As far as the latter procedure is concerned, we will also try to investigate the relevance of different policies with respect to the disclosure of information regarding the assessment of the reliability of each potential contractor.

Our main purpose is to analyse how different economic and institutional aspects can affect the effectiveness of each procedure. Obviously we will also deal with the issue of corruption and we will examine not only the impact of it on the efficiency of each procedure, but also the effect of each selection procedure on the probability that public officials choose to be corrupt.

We will show that contracting authorities can effectively take advantage of competitive negotiation only if a sufficient number of competing firms and a low risk of corrupted agreements exist. When only the first condition is not satisfied, negotiation could then be the most suitable alternative. If also the second condition does not hold, then the sole valid solution is to adopt standard auction mechanisms. Moreover, if the contracting authority is interested in reducing the probability of corruption, it should reveal its assessment of the reliability of the firms in transparent manner before asking them to submit their best and final offer. However, the more surprising conclusion is that, when the level of competition is very low and firm costs are largely heterogeneous, the best selection procedure may be a competitive negotiation with no disclosure of the assessment regarding the reliability of each bidding firm.

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[2] See for instance Latham (1994) and Egan (1998).

The rest of the work is organised as follow: in section 2 there is a review of the relevant literature. An analytical model is introduced in section 3. Sections 4 and 5 are devoted to the study of the outcomes associated to each selection procedure, with and without corruption. Section 6 summarizes the main results. Section 7 concludes.

2. Related literature

In economic literature there has been a long-standing debate on the comparison between auctions and negotiations. In a seminal paper, Bulow et *al*. (1996) demonstrated that an auction with *n+1* bidders and no reserve price gives the auctioneer a higher expected utility than the best negotiation mechanism in the presence of only *n* candidates. Actually, this conclusion does not support a preference for auctions as compared to negotiation. Its main indication is that it is more important for contracting authorities to increase competition, rather than to raise their negotiation power. However their model assumes that the auctioneer is ex-ante indifferent to the identity of the contractor. Conversely, we will hypothesise that bidding firms differ with regard to their reliability. For this reason our results will be different.

Manelli et *al.* (1995) analyse a model in which bidders differ with respect to their quality, an aspect that is assumed to be unobservable and unverifiable for the auctioneer. Within this framework, they show that an auction is optimal only in the presence of very limitative conditions, while a sequential offer procedure, that they call negotiation, is generally optimal. Also in our model the reliability of the firms is assumed to be unverifiable for a third party, and consequently, the contract price cannot be contingent on its value. However, and differently from their paper, we hypothesise that the procurer can ex-ante assess the reliability of the competing firms and the aim of our analysis is to focus on the best use of such information.

Another strand of literature that is closely related to our work is represented by all the papers dealing with the issue of multidimensional auctions. In fact, competitive negotiation can be seen as an auction with a bidimensional awarding criterion. The main part of the said literature adopts an optimal mechanism approach in order to define the optimal scoring rule. The most significant result achieved by different authors in a variety of environments is that quality must be undervalued with respect to price (Che, 1993; Naegelen, 2002). We will choose to follow a different hypothesis, assuming that the procurer adopts a scoring function that is equal to her welfare function. However, in certain sections of the paper, we will show how she can improve her welfare by modifying the scoring rule.

Some recent papers (Rezende, 2004; Gal-or et *al*., *forthcoming*; Doni et *al.*, 2007) have investigated the relevance of the information policy when the quality of firms is buyer's private information. The main result is that the optimality of a specific information policy depends on the ratio between the level of firms' cost heterogeneity and the weight assigned to their quality. We will extend this analysis by focusing on the relevance of the information policy when the agent who assesses the reliability of the firms can choose to be corrupt.

In Ganuza (1996) and Engelbrecht-Wiggans et *al*. (2005) we can already find a comparison between price auctions and multidimensional auctions. We will make use of an important result contained in the second of these papers, according to which multidimensional mechanisms are preferable only in the presence of a sufficient number of bidders.

Lastly, with regard to the issue of corruption, our model is closely related to the recent papers by Celentani et *al*. (2002) and Burguet et *al*. (2004). In fact, both these models consider the case in which a procurer delegates an agent to evaluate firm quality and the agent can manipulate his assessment in exchange for a bribe. The former paper assumes that the agent has to choose whether to be corrupt or not, and the analysis is devoted to a study of the interaction between competition and the probability of corruption. Their main result is that these two elements may be positively correlated. Conversely, Burguet et *al*. (2004) completely disregard the problem of the agent's choice and focus their attention on the efficiency of the selection mechanism when the agent is corrupt. Differently from Celentani et *al*. (2002), who assume that the agent is randomly matched to a firm, these authors hypothesise that the agent organises a bribery competition. They then prove that the final winner could not be the lowest cost firm.

We will try to combine various elements of these two models. We will assume that the agent can run a bribery auction and in this way we can calculate the maximum expected bribe associated to each different procedure. At the same time we hypothesise that the agent can choose whether to be corrupt or not. We can thus examine not only how his decision affects the outcome of each procedure, but also how different procedures influence the probability of corruption.

3. The model

We consider a model with $n + 2$ players: a procurer (P), her agent (A) and n competing firms. All the parties are assumed to be risk-neutral. P wishes to choose a firm to which to delegate the realization of a project. The welfare that P can obtain by contracting with a generic firm i is:

$$W = Q - p_i - (1 - r_i)K, \quad \text{where:} \tag{1}$$

Q represents the exogenous and fixed level of quality of the project;

p_i is the price contracted with firm i. The way in which this price is defined depends on the type of selection procedure adopted by P.

r_i is the degree of reliability of firm i. Note that a higher value of the indicator is associated with the greater reliability of the firm. We assume that $r_i \in [0;1]$, $\forall i=1, \ldots, n$. The assessment of a firm's reliability is completely discretional, and for this reason we assume that firms are ignorant of it. They have only a common probabilistic belief according to which r_i are independent and identical random variables distributed over the aforesaid interval. We define $G(x) = \text{Prob}\{(1-r) \leq x\}$.

K is a measure of the expected ex-post costs[3]. We assume its value is included in the interval $[0;\overline{K}]$ and it is common knowledge.

Note that K is implicitly related to both the complexity of the contract and the power of the legal system to enforce the initial terms. In fact, it is common sense that the more complex a project, the higher the unforeseen costs. At the same time, for a given level of complexity, the weaker the enforcement power of the legal system is, the higher the expected ex-post costs are.

K can also be interpreted as the importance that P assigns to a different degree of reliability. In fact, it corresponds to her maximum willingness to pay in order to have a fully reliable firm as a contractor ($r = 1$), instead of an absolutely unreliable one ($r = 0$)[4].

We hypothesise that P does not possess the ability to evaluate the reliability of suppliers. For this reason, she has to rely on the judgment of A. We assume that A can decide to reveal his evaluation correctly or to propose a corrupt agreement to a firm. As in Celentani et *al*. (2002), we hypothesise that A's utility depends on the difference between the expected bribe and the sunk cost of being corrupt:

[3] Actually, the product $(1-r)K$ can represent a monetary measurement of both an increase in the final costs and a reduction in the quality of the realised project.

[4] In Appendix 1 we illustrate a more detailed model and we show that the weight to assign to the subjective assessment of firms' reliability should depend also on the type of contract adopted. We confirm there the intuition of Bajari et al. (2006), according to which the relevance of the reliabiity factor increases when the contractual agreement is governed by a cost-plus scheme.

$U = \mathrm{E}[B] - \beta$;

without loss of generality, we assume that β is drawn from the uniform distribution on $[0; \overline{\beta}]$. Consequently we can identify the probability that A decides to be corrupt as:

$\phi = \min\ [\mathrm{E}[B]/\overline{\beta}\,;1]$.

The n potential candidates differ, depending on their production costs and on their reliability. The expected profit of the selected firm is independent of its actual reliability, and is equal to the difference between the contracted price and its expected costs.

$\pi_i = p_i - C_i$

We hypothesise that P and A are not acquainted with the firms' costs. Their beliefs are represented by independent random variables distributed according to the generic distribution F(C) over the closed interval $[\underline{C}\,; \overline{C}]$, and we define $\Delta = \overline{C} - \underline{C}$ as a measure of firms' cost heterogeneity. Conversely, we assume that every firm is informed not only as to its own costs, but also regarding the costs of its competitors.

We define also the valuation v_i of a generic firm i as the maximum P's welfare that it can provide under the constraint of a non-negative profit:

$v_i = Q - C_i - (1 - r_i)K$.

For every generic random variable of the model, x, we define $x_{(k)}$ the *k-th* largest of n independent samples. We introduce the variable δ that indicates the cost differential of the two lowest cost firms. Formally: $\delta = C_{(n-1)} - C_{(n)}$. Moreover, we define the concept of "efficient procedure" as the mechanism that guarantees the selection of the firm with the highest valuation, $v_{(1)}$. In fact this choice maximises the total surplus associated to the contract.

We assume that C_i and r_i are not correlated; moreover, we let their distributions satisfy the classic regularity condition:

$$\frac{\partial}{\partial x}\left[\frac{H(x)}{h(x)}\right] \geq 0,\ \forall H = G,F.$$

Furthermore, we hypothesise that the value of the project is sufficiently large with respect to both the degree of asymmetric information on the players and the level of contractual complexity. More specifically, our assumptions (*i* and *ii*) enable us to be sure that, in the case of negotiation between P and a generic firm, there is not a positive probability that the contract is not signed, while assumption *iii* guarantees that P's welfare is always positive, even when A chooses to be corrupt. Formally:

Assumption 1: Q is so large that the following conditions hold with certainty:

i) $Q > \overline{C} + \overline{K} + \frac{F(\overline{C})}{f(\overline{C})}$ *ii)* $Q > \overline{C} + \overline{K} + \frac{G(1)}{g(1)}\overline{K}$, *iii)* $Q > \overline{C} + 2\overline{K} + \overline{z}$,

where $\overline{z} = \left\{ \mathrm{E}\left[\frac{\delta}{2} - \frac{\delta^2}{18K} \right] \middle| \delta < 3\overline{K} \right\}$.

In view of the fact that we want to analyse the welfare achievable by P using different selection procedures, we will compare three different mechanisms: Negotiation (Ne), Auction (Au) and Competitive Negotiation (CN). In the first kind of procedure we assume that P's choice is mainly addressed to the selection of the most reliable firm. Conversely, in the second mechanism P commits herself to choose the contractor only on the basis of the bids submitted, disregarding completely the different degree of reliability of each firm. The last procedure represents a hybrid mechanism: P invites each firm to submit a bid, and commits herself to choosing the firm that maximizes a pre-announced scoring function, by weighing both the price and the reliability factor. In this specific mechanism, we will also investigate how the final outcome can be influenced by the policy regarding the disclosure of information about A's judgments. More specifically, we will analyse two cases: the concealment (CN-C), or the public revelation (CN-P) of the said information.

We can illustrate each procedure more in depth by means of an explicit description of the timing of the game. The first four steps are common to any procedure:

1. P announces the rules of the selection procedure;
2. Nature chooses ($r_1, \ldots, r_n$), ($C_1, \ldots, C_n$) and β;
3. A is informed about β, and then decides privately whether to be honest or corrupt;
4. A learns privately the reliability of each firm, while each firm is publicly informed about both its production costs and the costs of its competitors.

Note that we differentiate our analysis by Celentani et *al*. (2002) by assuming that the mechanism is announced before A has chosen whether to be corrupt or honest. In this way we can analyse whether different procedures are more or less vulnerable to corruption. In fact, the expected bribe that can be achieved in each mechanism is different and this fact can affect A's choice.

The subsequent steps differ depending on the selection procedure adopted and on A's choice and we will describe these in the following two sections[5].

[5] A synthetic scheme with the timing of the game for each selection procedure is reported in the appendix 2.

4. The outcome of each procedure in the "honest-agent" case

In this section we illustrate the subgame that characterises each procedure when A has chosen to honestly report his evaluation with regard to the reliability of each firm. In order to make a first comparison, we need to calculate P's expected pay-off for different types of procedures.

4.1.Negotiation

When A has decided to be honest, the extensive form of the Ne procedure is:

5. A indicates to P the firm with the greatest reliability;
6. P negotiates the price with the selected firm: with probability λ P makes the firm a take-it-or-leave-it (TIOLI) offer, and with probability $1-\lambda$ the firm makes P a TIOLI offer.

We have chosen to model the negotiation on the price as a reduced form game, following the description of the renegotiation stage in Bajari et *al.* (2001). λ is a parameter that varies between 0 and 1 and can be interpreted as P's negotiation power. We assume this factor to be exogenously determined by the institutional context. The disagreement pay-offs of each player are normalised to 0. Thanks to assumptions 1.i and 1.ii we can state the following lemma:

Lemma 1: when the Ne procedure is adopted, in the negotiation stage, P makes a TIOLI offer equal to $\overline{C}$ while the selected firm proposes a TIOLI offer equal to $Q-K$.

Proof: see the appendix 3.

Corollary 1: in this case the expected pay-off of P is (the subscript H indicates we are dealing with the "honest-agent" case):

$$E[W_H^{Ne}] = \lambda(Q - \overline{C} - K) + E[r_{(1)}]K, \quad [2]$$

where $E[r_{(1)}]$ is the expected reliability of the most reliable firm.

Proof: see the appendix 3.

4.2.Auction

When the Au procedure is adopted, A plays no role because the awarding rule completely disregards the reliability factor. Therefore the timing of the game in this case is:

5. A plays no role;

6.1 all the competing firms submit a price;

6.2 P selects the firm offering the lowest bid.

Given the assumption of perfect information among firms as to their production costs[6], it is easy to derive the following conclusion:

Lemma 2: when the Au procedure is adopted, the contract is always awarded to the lowest cost firm; the expected price for P is equal to the costs of the second lowest cost firm.

We omit a formal proof because this is the classic outcome of Bertrand competition on price between cost heterogeneous firms.

Corollary 2: in this case P's expected welfare is:

$$E[W_H^{Au}] = Q - E[C_{(n-1)}] - K + E[r]K. \qquad [3]$$

Proof: see the appendix 3.

4.3.Competitive negotiation

When A has decided to be honest, the extensive form of the CN procedure is:

5. A reveals to P the level of reliability of every competing firm;

6.1 all the firms are publicly informed about the reliability evaluation;

6.2 every firm submits an economic bid p;

6.3 P awards the contract to the firm that maximises the pre-announced scoring function and the winning firm receives a price equal to its bid.

We start by assuming that, in this kind of procedure, P decides to make a public revelation regarding the ranking of the firms with respect to their reliability[7]. This choice enables us to compare the CN procedure with its alternatives under very general assumptions. At the end of this section we will focus on the outcome that P may achieve if she chooses not to reveal this information.

Note that after the revelation of A's judgements, all the firms have complete information regarding the valuation of their competitor. It is then simple to deduce the following lemma:

Lemma 3: in CN-P procedure, the highest valuation firm $v_{(1)}$ wins the auction matching the valuation of the second highest valuation firm, $v_{(2)}$.

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[6] In every bidding game present in the model we assume that possible ties are broken by means of a Vickrey auction; in this way we can disregard problems related to the existence of an equilibrium in the presence of perfect information between bidders.

[7] In Doni et *al*. (2007) it is shown that the outcome of a first score auction with public revelation of P's private information is coincident with the outcome achievable by means of a second score auction and any sort of information policy. Note however that in this context a second score auction is not equivalent to an English auction. Iin fact, as showed by Wambach (2002), an English auction on the price in the presence of an unknown awarding criterium can induce a tacit collusive agreement between the competing firms.

We omit a formal proof because also this result is based on the classic argument of Bertrand competition. However, in this case firms compete in scores on the basis of the information regarding their valuations.

Corollary 3: in this case P's expected welfare is:

$$\mathrm{E}[W_H^{CN}] = \mathrm{E}[v_{(2)}].$$

Proof: this conclusion is a direct consequence of Lemma 3.

4.4. Comparisons of the selection procedures in the "honest-agent" case

We start by comparing P's expected welfare when the Ne or Au procedures are adopted. By observing equations [2] and [3], it easy to note that the Ne and Au procedures differ notably in their results. In fact the former procedure benefits P by making sure that the most reliable firm is selected. Conversely, the latter procedure guarantees only an average level of reliability, but helps P to reduce her asymmetric information on the costs of the competing firms. More formally, the following equation shows the difference in P's expected welfare in the two cases:

$$\mathrm{E}[W_H^{Ne}] - \mathrm{E}[W_H^{Au}] = (\mathrm{E}[r_{(1)}] - \mathrm{E}[r])K - (1-\lambda)(Q - \overline{C} - K) - (\overline{C} - \mathrm{E}[C_{(n-1)}]). \qquad [4]$$

It is then simple to derive the following conclusion:

Proposition 1: the Ne procedure is more convenient with respect to Au:

i) the higher is the value of λ;

ii) the higher is the value of K;

iii) the lower is the value of Δ.

Proof: see the appendix 3.

The first part of this proposition is quite obvious and tell us that P must possess sufficiently strong negotiation power in order to take advantage of the Ne procedure. Part ii confirms the empirical result of Bajari et *al*. (2006) from a theoretical point of view: i.e., when the level of contractual complexity is very high, a negotiation procedure can be better than an auction. However, the last part points out that the Au procedure may be preferable when the firms' costs are largely heterogeneous.

It is worth noting that an increase in the number of candidates has no clear effect on equation [4]. In fact, a higher level of competition increases the expected reliability of the selected firm when the Ne procedure is adopted. At the same time, however, it reduces the expected price when the Au mechanism is adopted. We can therefore state a proposition in strong contrast with the conclusions of Bulow et *al*. (1996):

Proposition 2: even if P had not all the negotiation power, she could obtain a higher expected welfare by negotiating in the presence of *n* candidates than by running an auction with *n+1* competing firms.

Proof: see the appendix 3.

The result of Bulow et *al.* (1996) is different because they implicitly consider firms with the same level of reliability.

In order to compare these two procedures to the CN-P mechanism we need to introduce a new technical concept. Given a generic random variable x, define n_x* as the smallest n such that $E[x_{(2)}] > E[x]$. For the properties of order statistics, $n_x* \geq 3$ whatever the distribution of x is[8].

It is now possible to state the following proposition:

Proposition 3: i) $n < \max [n_v*, n_C*] \Leftrightarrow E[W_H^{Au}] > E[W_H^{CN-P}]$;

ii) $n \to \infty \Rightarrow E[W_H^{CN-P}] - E[W_H^{Ne}] > 0$;

iii) only CN-P procedures guarantees an efficient selection.

Proof: see the appendix 3.

This proposition points out some important results. The first one is that if P chooses to run a competitive procedure, but the number of firms is very low, she could then find it convenient to disregard the reliability factor. Conversely, part ii emphasises that when competition is sufficiently high, CN-P mechanism is preferable to both the Ne and Au procedures. Lastly, part iii states that only the CN-P procedure ensures the selection of the firm which maximises the social surplus of the contract.

Therefore, if we could take the honesty of A for granted, CN-P would be the best procedure, providing that a sufficient number of candidates is present.

It is important to note that until now we have represented P's strategy in the CN-P procedure in a rather naïve way. In fact when P runs a competitive negotiation, she can strategically choose both the weight to assign to the differences in firms' reliability and the policy regarding the disclosure of her private information. For instance, Shachat et *al*. (2002) shows that the optimal bias in favour of the most reliable firm should always be lower than the actual benefit that P receives from its selection. Conversely, Doni et *al*. (2007) investigate the optimal information policy when P is unable to adopt an optimal awarding rule. According to their analysis, P may find it convenient to conceal her private information with regard to the reliability of the competing firms when the ratio between the firms' cost heterogeneity and the

[8] If E[median(x)] $\geq$ E[x], then $n_x* = 3$; note that this condition is valid for all the symmetric distributions.

weight assigned to reliability is high, with very high probability. Otherwise, a public revelation of this information is preferable for P.

What is interesting is that both these papers analyse a model involving only two firms and this fact enables us to deduce that the CN mechanism can be better than the Au and Ne procedures also in this case. For this to be true, it is sufficient that P chooses optimally either the awarding criterion, or the information policy.

In the following section we will investigate the outcome of each procedure in the case that A decides to be corrupt. In the subsection regarding CN we will also examine more thoroughly how the information policy can affect the choice of A with regard to the probity of his behaviour.

5. The outcome of each procedure in the "corrupt-agent" case

We have already said that A has no decision power when P runs an auction; therefore we will not consider this particular procedure in this section. We will describe the subgames associated to the Ne, CN-P and CN-C mechanisms. We follow closely Burguet et *al* (2004) in order to try to capture some of the potential dynamics of corrupt agreements. As in their model, we adopt the idea that A is able to organise a sort of competitive auction. We assume that A asks to each firm to propose a bribe in exchange for the promise that he will use all of his manipulative power in order to favour the "winning" firm. In our model the maximum manipulation power is equal to K, that indicates both the level of contractual complexity and P's maximum willingness to pay for awarding the contract to a fully reliable firm instead of selecting an absolutely unreliable one. This is coherent with the existing opinion that the higher the complexity of the contract is, the higher the discretionary power of public officials will be. In the following subsection we will show how different selection mechanisms affect both the probability that A decides to be corrupt and P's expected welfare.

5.1. Negotiation

When A decides to be corrupt, the extensive form of the Ne procedure is:

5.1 A runs a sealed bid auction on the bribe B;

5.2 the firm that outbribes all its opponent is selected and is evaluated as being fully reliable ($r=1$);

6. P and the selected firm negotiate the price as in the "honest-agent" case.

In this case the bribe that each firm is disposed to pay is a direct function of the profit it can realize in the case it is selected and assessed to be fully reliable. The profit of a generic firm i is equal to the difference between the price that can be achieved in the negotiation with P and its costs. The difference between this and the "honest-agent" case is that the selected firm now knows it has been judged to be fully reliable. Consequently, it can demand a higher price during the negotiation stage. We can then deduce the following lemma:

Lemma 4: in the Ne procedure, when A is corrupt, the lowest cost firm will out-bribe all its opponent offering a bribe equal to $B(C_{(n)}) = \lambda\overline{C} + (1-\lambda)Q - C_{(n-1)}$. The expected price of the contract negotiated between P and the selected firm is equal to $\lambda\overline{C} + (1-\lambda)Q$.

Proof: see the appendix 3.

This outcome is the result of bribery competition in the Bertrand fashion. Note that the reliability factor plays no role in the selection of a specific firm.

Corollary 4: if P adopts Ne procedure:

$$\phi^{Ne} = \min\left[\frac{(1-\lambda)Q + \lambda\overline{C} - \mathrm{E}\left[C_{(n-1)}\right]}{\overline{\beta}};1\right];$$

$$\mathrm{E}[W_C^{Ne}] = \lambda(Q - \overline{C} - K) - (1 - \lambda - \mathrm{E}[r])K. \qquad [5]$$

Proof: see the appendix 3.

By comparing equations [2] and [5] we can observe that when A is corrupt P's expected welfare diminishes. There are two reasons for this: in the first place, there is a loss equal to $(1-\lambda)K$ due to an increase in the negotiated price. Secondly, the selected firm now has only an average level of reliability instead of the expected value of the first order statistics of r.

5.2.Competitive negotiation

In the CN procedure P selects a firm on the basis of A's judgments, but also on that of bids submitted by firms. As stressed by Burguet et *al*. (2004), the outcome of the bribery competition will be greatly affected by the timing of the game. These authors hypothesise that firms have to propose a bribe to A at the same time that they make their bid regarding the price. Consequently, the firms' bids are formulated before knowing A's message to P. In a footnote they also consider the possibility that the reliability of each firm has to be evaluated and publicly announced by A prior to the submission of economic bids. Note that the former case coincides with the concealment of A's assessments, while the latter case is equivalent to the public revelation of such information.

In this section we will consider both cases. We can start by analyzing the case of public revelation. In this case, the timing of the subgame is:

5.1 A runs a sealed bid auction on bribe B;

5.2 the firm i which outbribes all its opponent is favoured; it pays to A the proposed bribe in exchange for being evaluated as fully reliable (r_i = 1), and for a declaration of not reliability with regard to its opponents ($r_j = 0$);

6.1-3; the competitive auction takes place by following the same steps of the "honest-agent" case.

We can disregard what might happen if A uses his manipulative power in favour of a firm that is not awarded the contract. In fact, in our model there is only one Nash-equilibrium in which the favoured firm always wins the auction. As it has been thoroughly explained by Burguet et *al*. (2001) in a previous version of their paper, in this kind of game the lowest cost firm can always capture A. The equilibrium of this game can now be synthesised in the following lemma:

Lemma 5: in P adopts the CN-P mechanism, i) if $K \leq \delta$, then in the unique Nash-equilibrium the lowest cost firm wins the bribery competition by submitting a bribe arbitrarily close to 0; then it offers a price equal to $C_{(n-1)} + K$ and it will be selected as contractor. ii) If $K > \delta$, then in the unique Nash-equilibrium the lowest cost firm wins the bribery competition by submitting a bribe equal to $K–\delta$; then it proposes a price equal to $C_{(n-1)} + K$ and it is selected as contractor.

Proof: see the appendix 3.

It is interesting to note that in the model of Burguet et *al*. (2004) the existence of a corrupt agent does not prevent an efficient selection when P adopts a public information policy. In fact the authors show that the contract is always awarded to the lowest cost firm. However, according to our model this outcome might not be efficient: in fact the lowest cost firm might not be the one which maximises the social surplus of the contract.

Corollary 5: if P adopts the CN-P mechanism:

$$\phi^{CN-P} = \min \left[\frac{\mathrm{E}\left[(K-\delta)\middle|\delta<K\right]\mathrm{Prob}\{\delta<K\}}{\bar{\beta}};1 \right];$$

$$\mathrm{E}[W_C^{CN-P}] = Q - \mathrm{E}[C_{(n-1)}] - 2K + \mathrm{E}[r]K.$$

Proof: see the appendix 3.

In the alternative case in which firms must submit a price without being informed as to A's judgments, the game reaches a different equilibrium. In this part, we closely follow the model of Burguet et *al*. (2004); for this reason we will limit our attention to the case in which n = 2.

The timing now is:

5.1 A asks each firm to submit a bribe *B* simultaneously with the contract price *p*;

5.2 A declares $r_j = 0$ for all the firms except firm *i* that has offered B_i s.t. $p_i < p_j + K$; firm *i* pays the proposed bribe in exchange for A's $r_i = 1$ evaluation.

6. P awards the contract to the firm that maximises the pre-announced scoring function and the winning firm receives a price equal to its bid.

Note that, in this case, it is implicitly assumed that A can know the price bid of each firm before declaring his judgments with respect to their reliability. Usually, in public procurement the awarding committee cannot open the envelope containing the economic offer before having publicly announced the score assigned to the technical proposal. This expedient is evidently not sufficient to avoid the possibility of corrupt agreements. However, it is clear that procedures similar to competitive negotiation are more vulnerable. In fact, public officials are able to obtain information regarding the economic aspects of each bid during the negotiation stage. In the following lemma we report the equilibrium of this game as illustrated in Burguet et *al*. (2004):

Lemma 6: when P adopts the CN-C mechanism:

i) if $K \leq \delta$, then it is an equilibrium for the highest cost firm to play a mixed strategy, offering a bribe uniformly distributed in the interval $[0,2K]$ and then bidding a price equal to its bribe more its costs. On the other hand the lowest cost firm proposes a bribe arbitrarily close to 0 and a price equal to $C_{(n-1)} - K$[9]. The lowest cost firm wins with probability 1;

ii) if $K > \delta$, then it is an equilibrium for both the firms to choose a bribe uniformly over the interval $[0,2K]$, and submit a price equal to:

$$\begin{cases} p\left(C_{(n)}\right) = C_{(n)} + \frac{1}{3}\delta + B\left(C_{(n)}\right) + K \\ p\left(C_{(n-1)}\right) = C_{(n-1)} - \frac{1}{3}\delta + B\left(C_{(n-1)}\right) + K \end{cases}$$

the lowest cost firm wins with probability $\frac{1}{2} + \frac{\delta}{6K} < 1$.

Proof: this lemma is a direct application of Proposition 1 of Burguet et *al*. (2004) to our model.

The next corollary shows A's expected bribe and P's expected welfare in this case.

Corollary 6: if P adopts CN-P mechanism:

[9] Actually, as explained by Burguet et *al*. (2004) in this case there are multiple equilibria. We follow them in assuming that, ceteris paribus, all the firms have a strict preference for offering the lowest possible bribe.

ϕ^{CN-C} = min $\left[\frac{\mathrm{E}\left[K \mid \delta < 3K\right]\mathrm{Prob}\{\delta < 3K\}}{\overline{\beta}}; 1\right]$;

E[W_C^{CN-C}]=Q − E[$C_{(n-1)}$] − K + E[r]K +

+E[$K \mid \delta \geq 3K$]Prob{$\delta \geq 3K$}+E[($2K$+z) $\mid \delta < 3K$]Prob{$\delta < 3K$}

where $z = \frac{\delta}{2} - \frac{\delta^2}{18K}$.

Proof: see the appendix 3.

5.3.Comparisons of the selection procedures in the "corrupt-agent" case

We can now analyse and compare the outcome associated with each procedure in the case that A decides to be corrupt.

Proposition 4: i) ϕ^{Ne}, ϕ^{CN-P} and ϕ^{CN-C} are all increasing in n; ii) both ϕ^{CN-P} and ϕ^{CN-C} are increasing in K, iii) ϕ^{Ne} is increasing in Δ.

Proof: see the appendix 3.

The first part of this proposition is quite surprising: it points out that the probability that A becomes corrupt is increasing in the level of competition, whatever the mechanism adopted is. Therefore, also our model is coherent with the result obtained by Celentani et *al*. (2002), according to whom in certain cases corruption may have a positive correlation to competition. The second part confirms the existing opinion that corruption is more probable when the contract to be awarded is incomplete and public officials must exert a largely discretionary power in selecting the contractor. The last part states that when the Ne procedure is adopted the probability that A becomes corrupt is increasing in the extent of P's asymmetric information.

Proposition 5: i) $\phi^{CN-C} \geq \phi^{CN-P}$, $\forall$K; ϕ^{CN-P} > (<) ϕ^{Ne} if Δ/K is sufficiently low (high).

Proof: see the appendix 3.

This proposition shows that a transparent announcement of A's evaluation prior to the request for a final offer might reduce the risk of a corrupt agreement between A and a firm.

Proposition 6: i) E[W_C^{CN-P}] $>$ ($<$) E[W_C^{CN-C}] if Δ/K is sufficiently low (high); ii) E[W^{Au}] is surely higher than both E[W_C^{Ne}] and E[W_C^{CN-P}], whatever the values of Δ and K are.

Proof: see the appendix 3.

The first result calls to mind the comparison between the two different information policies in the "honest-agent" case. As stated in the first point of Proposition 6, when the maximum cost differential is very low with respect to the weight assigned to the reliability factor, P can benefit from disclosing A's judgments, even in the case A decides to be corrupt. On the other hand, when the complexity of the contract is minor compared to the potential cost differential between firms, P can improve her expected welfare by concealing A's evaluation, even if A chooses to be corrupt[10]. Lastly, the second point states that if the corruption of A is highly probable, P should run a simple auction and avoid both the Ne and CN-P mechanisms.

Table 1 – Synthesis of the main results

	P's expected welfare in the "Honest-agent" case	P's expected welfare in the "Corrupt-agent" case"	Probability that A is corrupt
Negotiation ($\lambda = 1$)	$Q - \overline{C} - K + \mathrm{E}[r_{(1)}]K$	$Q - \overline{C} - K + \mathrm{E}[r]K$	$\mathrm{Prob}\{\beta < \overline{C} - \mathrm{E}[C_{(n-1)}]\}$
Auction	$Q - \mathrm{E}[C_{(n-1)}] - K + \mathrm{E}[r]K$	$Q - \mathrm{E}[C_{(n-1)}] - K + \mathrm{E}[r]K$	0
Competitive Negotiation with Public revelation	$\mathrm{E}[v_{(2)}]$	$Q - \mathrm{E}[C_{(n-1)}] - 2K + \mathrm{E}[r]K$	$\mathrm{Prob}\{\beta < \mathrm{E}[(K-\delta)\mathrm{Prob}\{\delta < K\}]\}$
Competitive Negotiation with Concealment	Better (worse) than CN-P if Δ/K is sufficiently high (low)	$Q - \mathrm{E}[C_{(n-1)}] - K + \mathrm{E}[r]K + \mathrm{E}[K\ \mathrm{prob}(\delta \geq 3K)] - \mathrm{E}[(2K+z)\mathrm{prob}(\delta < 3K)]$	$\mathrm{Prob}\{\beta < \mathrm{E}[K\ \mathrm{Prob}\{\delta < 3K\}]\}$

6. Discussion of the results

All the results achieved in sections 4 and 5 with regard to P's expected welfare and to the probability that A decides to be corrupt can be found in Table 1.

In this section we will try to synthesise the main comparisons between the different procedures. We will focus on the relevance of some economic or institutional factors, such as the number of candidates, the weight assigned to the reliability of firms, the degree of heterogeneity of production costs, A's sunk costs of being corrupt. More specifically, we will disregard the relevance of P's negotiation power, and assume that $\lambda = 1$. We know that a lower value of λ will diminish the effectiveness of the Ne procedure. As far as all the other factors are concerned, we will make some distinctions with regard to:

i) the level of competition: $n = 2/ n = 6$;

ii) the ratio Cost Heterogeneity/Reliability Weight: very low/very high;

iii) the value of $\overline{\beta}$: very low/very high.

[10] At the same time, when K is low, the expected bribe also tends to 0: for this reason, it is probable that A will perform his task honestly.

We know that the value of $\overline{\beta}$ influences the probability that A's decides to be corrupt: in fact if $\overline{\beta}$ were equal to 0 A would surely choose to be corrupt; conversely, if $\overline{\beta}$ tended to infinite, A would then decide to be honest with very high probability. We also recall that the analysis of the CN-C procedure is valid only for the case in which $n = 2$.

In figures 1-4 we can distinguish four different cases[11]:

I. in observing figure 1 we can see that there is a high probability that P can benefit from using the Ne procedure in this specific case, on condition that the sunk costs of being corrupt are sufficiently high. In the contrary case, the best choice for P is to adopt the Au procedure.

II. in this case, as shown by figure 2, the Ne and CN-P mechanisms are unquestionably less recommendable than the Au procedure. The CN-C mechanism can represent the best selection rule if the level of heterogeneity of the firms and the costs of corruption are sufficiently high.

III. in this case, we know from proposition 3 that if A decides to be honest the CN-P mechanism is better than the Au procedure. The Ne procedure is also probably preferred to Au, given the contents of proposition 1. However, as we can see in figure 3, in the corrupt-agent case the ranking is completely reversed: Au becomes the best alternative, and CN-P the worst one. Therefore, what is really relevant in this case is the probability that A will choose to be corrupt. By observing table 1 we can deduce that ϕ(CN-P) > ϕ(Ne) > ϕ(Au) = 0, $\forall \overline{\beta}$. Consequently, it is very difficult to identify the best alternative for P in this specific case. The CN-P mechanism will be preferred when the number of bidders and the value of $\overline{\beta}$ are sufficiently high. In the contrary case, the Ne or Au procedures might represent better alternatives.

IV. in observing figure 4 we can see that CN-P is definitely preferable to the Ne procedure in this case. As far as the comparison between the CN-P and Au mechanisms is concerned, we can state that the higher the number of firms or the value of $\overline{\beta}$, the higher P's expected welfare by adopting the CN-P, instead of the Au mechanism.

[11] For each selection procedure, the point on the left (right) represents a higher (lower) value of $\overline{\beta}$.

Figure 1
Case I: n = 2; Δ/K low

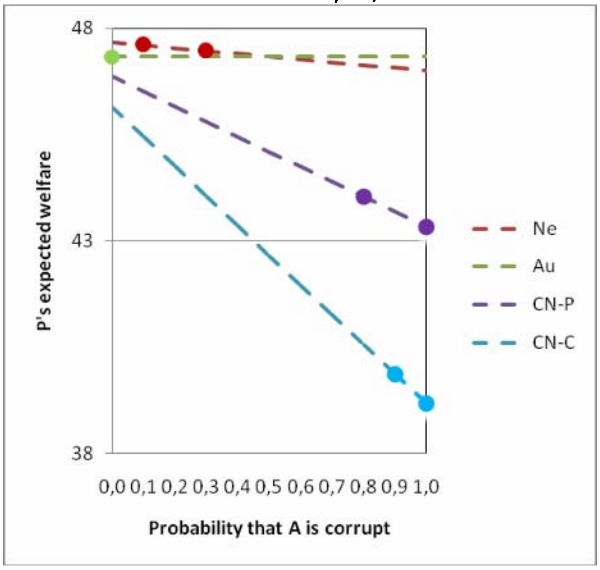


Figure 2
Case II: n = 2; Δ/K high

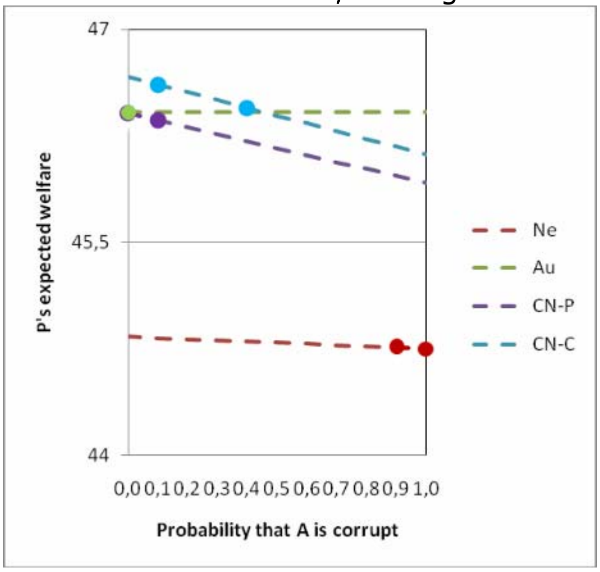


Figure 3
Case III: n = 6; Δ/K low

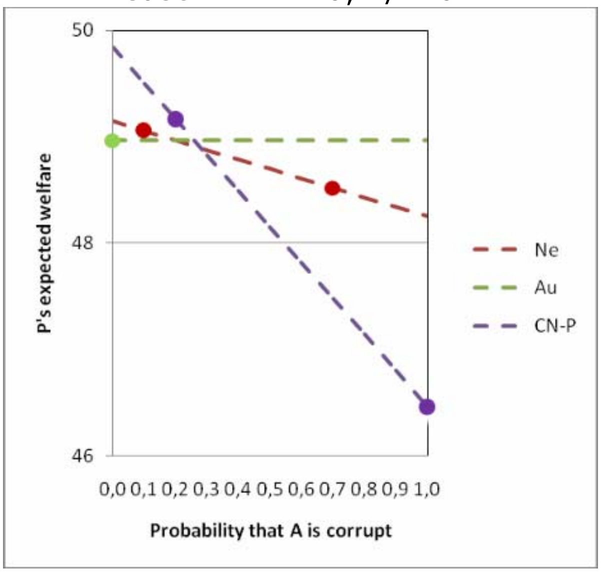
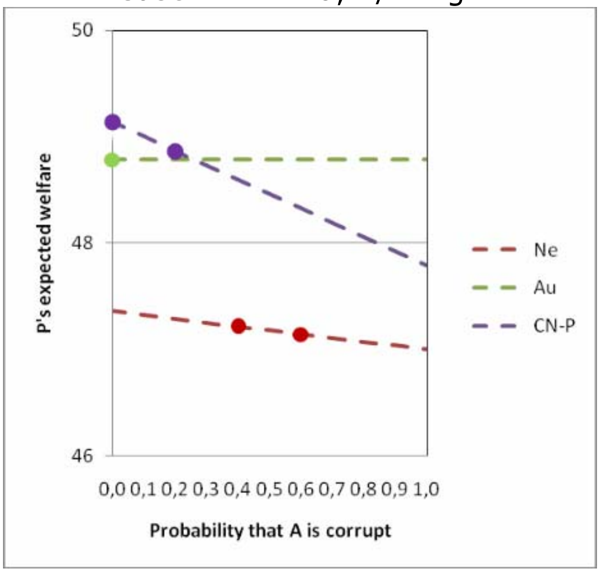


Figure 4
Case IV: n = 6; Δ/K high



We can conclude this section by emphasising the necessary conditions for CN to be a convenient selection procedure: i) there must be a serious level of competition in the sector (two is not enough, and three is barely sufficient); ii) the institutional environment should effectively discourage agents from being corrupt. In this sense, the public disclosure of A's judgments before the submission of final bids from firms could be an useful rule for promoting a greater transparency and for reducing the probability of corruption.

7. Concluding remarks

In this work we have compared different procedures aimed at selecting a contractor: i.e. auction, negotiation and competitive negotiation. Our main conclusions are that none of these procedures can be considered as absolutely better than its alternative and that their suitability depends on various economic and institutional factors: e.g. the level of competition, the enforcement power of the legal system, the complexity of the contract and the sunk costs incurred by public officials for being corrupt.

In this final section we want to use our results in order to give a possible explanation for the diffusion of different kind of selection procedures in different situations. We have said that, when we are in the presence of a small number of candidates and of a sufficiently low risk of corruption phenomena, the best procedure can probably be that of negotiation. This situation calls to mind the water sector in France, where the industry is highly concentrated and local authorities have always preferred not to base their choices on competitive mechanisms. We have also seen that when we are in the presence of a significant risk that public officials choose to be corrupt, the only suitable procedure is an auction mechanism. This conclusion resembles the situation in Italy, where the proliferation of corruption phenomena has induced the legislator to restrict greatly the flexibility of the selection procedures. Competitive negotiation probably originated in the United Kingdom because the institutional framework there can guarantee a sufficient level of competition and a low probability of corruption.

In our opinion, a vicious circle could be established in those situations in which negotiation and auction are the common procedures. For instance, we have seen that negotiation can be the best procedure when a sector is concentrated; but it is evident that incumbent firms are favoured by this selection mechanism because it can be very difficult to assess the reliability of new entrants. Therefore, we may have an equilibrium in which the negotiation procedure and a low level of competition sustain each other reciprocally. At the same time, if contracting authorities adopt only auction mechanisms, firms do not find it convenient to invest in reliability. Consequently in a situation of this kind, the total costs of a complex contract are usually very high. It might thus be better to adopt only standard contracts there, and to relinquish the benefits associated with innovative solutions.

However, the main message of the paper is that the adoption of competitive negotiation does not automatically ensure improvement in public welfare. In fact, in order to take advantage of its positive properties, a public authority is obliged to develop some specific policies. In the first place, the introduction of competitive negotiation does not automatically produce competition. It is then necessary that the public authority could actively guarantee an actual contestability in the awarding procedure of each contract. Secondly, corruption phenomena must be seriously limited.

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Appendix 1: A more detailed model

Assume that the assessment regarding the reliability of a specific firm *i* can be disentangled in two different judgments:

$\rho_i = \eta x_i + (1-\eta)a_i$ where $x_i, a_i, \eta \in [0;1]$:

a_i is an objective measure based on elements as financial standing, technical capability and so on; it can be considered common knowledge;

x_i corresponds to a subjective evaluation based on past performance, references, and so on; its assessment is a private information of the agent;.

η represents the degree of subjectivity entailed in the assessment of firms' reliability.

We can also try to include in our model a variable representing the type of contract; we hypothesise that procurer's welfare is equal to:

$$W_i = Q - p - (1-\gamma)(1 - \rho_i)K, \quad \text{[A1]}$$

while the expected costs for a specific firm can be represented as:

$$C_i = \theta_i + \beta a_i + \gamma(1 - \mathrm{E}[\rho_i])K, \quad \text{where:} \quad \text{[A2]}$$

θ_i is known by all the competing firms and unknown by the procurer,

β is common knowledge.

Note that if $\gamma = 1$ then we have a fixed price contract with perfect enforcement. In fact, in this case procurer's welfare is independent of the level of firm's reliability. Conversely, if $\gamma = 0$ then we are dealing with a cost plus contract with perfect enforcement because firm's profit are independent of the level of ex-post costs. In practice we think that the enforcement is never perfect, and consequently, whatever the contract adopted, both procurer's welfare and firm's profit are dependent on the extent of ex-post costs. So we let γ assume also values between 0 and 1. Obviously, if the contract is cost-plus (fixed price) the value of γ tends to be close to 0 (1).

Following the idea of Bajari et *al*. (2001) we can assume that the level of ex-post costs, *K*, depends on the type of contract adopted. According to their idea, for low (high) level of project complexity the amount of ex-post cost is higher (lower) with a cost-plus than with a fixed price scheme. In our model we can include this point assuming that the procurer chooses the type of contract which minimises the level of ex-post costs. Therefore we hypothesise that the level of ex-post costs depends on the optimal choice of the type of contract; formally:

$$K = K(\gamma^*)$$

We substitute the equation representing firms' reliability in equations [A1] and [A2]:

$$W_i = Q - p - (1-\gamma^*)(1 - (\eta x_i + (1-\eta)a_i))K(\gamma^*),$$

$$C_i = \theta_i + \beta a_i + \gamma^*(1 - (\eta \mathrm{E}[x_i] + (1-\eta)a_i))K(\gamma^*)$$

We can now define:

$$Q_i = Q - (1-\gamma^*)(1 - \eta)(1-a_i)K(\gamma^*)$$

$$(1-r_i)K = (1-\gamma^*)\eta(1 - x_i)K(\gamma^*)$$

In this way we obtain a perfect correspondence with the model analysed in the paper, with the unique difference that now firms are ex-ante asymmetric with respect to their quality and to their costs. However, this novelty does not imply any difference in firms strategies because they are perfectly informed about these parameters.

In this representation we can see that procurer should assign to her subjective evaluation ($1 - x_i$) a weight increasing in *K* and η and decreasing in γ. Moreover, the higher the value of γ, the higher will probably be the ratio between the extent of heterogeneity of firms' costs and the weight assigned to the subjective assessment of the reliability of the firms.

Appendix 2: The timing of the game for each selection procedure

<table>
<tr><th></th><th colspan="2">Negotiation</th><th>Auction</th><th colspan="2">Competitive Negotiation and public revelation of A’s judgements</th><th colspan="2">Competitive Negotiation and concealment of A’s judgments</th></tr>
<tr><td>1</td><td colspan="2">P announces that she will negotiate the price with the most reliable firm</td><td>P announces that she will run a first price sealed bid auction;</td><td colspan="4">P announces that she will run a first score auction where the scoring function to be maximised is: $Q - p - (1 - r)K$</td></tr>
<tr><td>2</td><td colspan="7">Nature chooses ($r_1, \ldots, r_n$), ($C_1, \ldots, C_n$) and β;</td></tr>
<tr><td>3</td><td colspan="7">A is informed about β , and then decides privately whether to be honest or corrupt;</td></tr>
<tr><td>4</td><td colspan="7">A learns privately the reliability of each firm, while firms are informed about the production costs of each of them;</td></tr>
<tr><td>5</td><td>A correctly indicates to P the firm with the greatest reliability;</td><td>1. A runs a sealed bid auction on the bribe B;
2. The firm that out-bribes all its opponents will be selected, and will be evaluated as being fully reliable ($r = 1$);</td><td>in this case A plays no role in the selection procedure;</td><td>A correctly reveals to P the level of reliability of every firm</td><td>1. A runs a sealed bid auction on bribe B;
2. the firm i which out-bribes all its competitors is favoured; it pays to A the proposed bribe in exchange for being evaluated as fully reliable (r_i=1), and for a declaration of unreliability with regard to its opponents ($r_j=0$);</td><td>1. A correctly reveals to P the level of reliability of every firm;
2. every firm must submit a price without knowing A’s judgements with regard to their reliability</td><td>1. A asks to each firm to submit a bribe B simultaneously with the contract bid p.
2. A declares $r_j = 0$ for all the firms except firm i maximising B_i s.t. $p_i < p_j + K$; firm i pays the proposed bribe in exchange for A’s evaluation $r_i = 1$.</td></tr>
<tr><td>6</td><td colspan="2">P negotiates the price with the selected firm: with probability λ P makes the firm a take-it-or-leave-it (TIOLI) offer, and with probability 1− λ the firm makes P a TIOLI offer.</td><td>1. all the competing firms submit a price;
2. P selects the firm offering the lowest bid.</td><td colspan="2">1. all the firms are publicly informed about the reliability evaluation;
2. firms submit the economic bid p;
3. P awards the contract to the firm maximising the pre-announced scoring function and the winning firm receives a price equal to its bid.</td><td colspan="2">P awards the contract to the firm maximising the pre-announced scoring function and the winning firm receives a price equal to its bid.</td></tr>
</table>

The shaded areas indicate the timing when the agent chooses to be corrupt

Appendix 3: Mathematical Proofs

Proof of Lemma 1: P chooses her TIOLI price offer p_P by maximising her expected welfare:

$$\max_{p_P} (Q - p_P - K)F(p_P),$$

where $F(p_P)$ is the probability that the costs of the selected firm are lower than the offered price. We differentiate with respect to p_P and we obtain:

$$\begin{cases} (Q - p_P - K)f(p_P) - F(p_P), & \text{if } p_P \leq \overline{C}, \\ -1, & \text{if } p_P > \overline{C}. \end{cases} \qquad [A3]$$

From assumption 1.i and the regularity condition regarding F_C we know that:

$$Q > K + C + F(C)/f(C),\ \forall C \in \left[\underline{C};\overline{C}\right].$$

Consequently, equation [A3] is surely higher than 0 and the solution to the maximisation problem is $p_P = \overline{C}$.

In the same way, A chooses his TIOLI price offer p_F by maximising his expected profit:

$$\max_{p_F} (p_F - C)G\left(\frac{Q-p_F}{K}\right),$$

We differentiate with respect to p_F and we achieve:

$$\begin{cases} G\left(\frac{Q-p_F}{K}\right) - (p_F - C)g\left(\frac{Q-p_F}{K}\right)\frac{1}{K}, & \text{if } p_F \geq Q - K, \\ 1, & \text{if } p_F < Q - K \end{cases} \qquad [A4]$$

Note that assumption 1.ii and the regularity condition imply that equation [A4] is surely negative. Therefore the optimal solution for the selected firm is to offer $p_F = Q - K$. *Q.E.D.*

Proof of Corollary 1: If P adopts the negotiation procedure, the expected reliability of the selected firm is equal to the expected value of the first order statistic of *r*. At the same time, from lemma 1 we know that when this procedure is adopted, the expected price is:

$$E[p_H^{Ne}] = \lambda\overline{C} + (1-\lambda)(Q - K).$$

We include these facts in equation [1] and developing we obtain:

$$E[W_H^{Ne}] = E[Q - p - (1 - r)K] = \lambda(Q - \overline{C} - K) + E[r_{(1)}]K. \qquad \textit{Q.E.D.}$$

Proof of Corollary 2: when P runs an auction she disregards the reliability of firms in the selection procedure, so the expected reliability of the selected firm is equal to the average level of reliability, E[*r*]. At the same time, from lemma 2 we know that the expected price for the procurer is equal to $E[C_{(n-1)}]$. We substitute these two results in equation [1] and we obtain:

$$E[W_H^{Au}] = E[Q - p - (1 - r)K] = Q - E[C_{(n-1)}] - K + E[r]K. \qquad \textit{Q.E.D.}$$

Proof of Proposition 1: differentiating equation [4] we obtain:

$$\frac{\partial\left\{E\left[W_H^{Ne}\right] - E\left[W_H^{Au}\right]\right\}}{\partial\lambda} = Q - \overline{C} - K,$$

$$\frac{\partial\left\{E\left[W_H^{Ne}\right] - E\left[W_H^{Au}\right]\right\}}{\partial K} = (E[r_{(1)}] - E[r]) + (1 - \lambda)$$

Given assumption 1, the first condition is surely positive. Given the assumption regarding λ and the properties of order statistics, also the second condition is always positive. Lastly, for the properties of order statistics we know that the distance between the upper bound of a distribution and the expected value of the *k-th* order statistic is always increasing in the size of the support of the distribution. Therefore the difference $\overline{C} - E[C_{(n-1)}]$ is increasing in Δ. *Q.E.D.*

Proof of Proposition 2: Assume that Δ goes to 0. In this case the last term of equation [4] is arbitrarily low, whatever the number of firms is. Consequently, if λ is sufficiently large:

E[W_H^{Ne} (n)] > E[W_H^{Au} (n)] = E[W_H^{Au} $(n+1)$]. *Q.E.D.*

Proof of Proposition 3: i) the first part of proposition 3 is an application of the proposition 6.D of Engelbrecht-Wiggans et *al*. (2005). In reality, they analyse a slightly different model, because according to their assumptions each firm is privately informed about its own costs, while their quality is known also by the procurer, but it is unknown by their opponent. Conversely, in our model, when P adopts a CN-P mechanism, both the quality and the costs of each firm are public information. However, we know that when bidders' valuations are IPV, the expected welfare of a first price (score) auction is equivalent to the expected value of the second highest bidder in case of both public and private information.

ii) From the properties of order statistics we know that:

$$\underset{n\to\infty}{Lim}\mathrm{E}\left[W_H^{CN-P}\right] = \underset{n\to\infty}{Lim}\mathrm{E}\left[v_{(2)}\right] = \bar{v} = Q - \underline{C}$$

$$\underset{n\to\infty}{Lim}\mathrm{E}\left[W_H^{Ne}\right] = \underset{n\to\infty}{Lim}\mathrm{E}\left[\lambda\left(Q-\overline{C}-K\right)+r_{(1)}K\right] = \lambda\left(Q-\overline{C}\right) + (1 - \lambda)K$$

By comparing the two equations is straightforward to state that:

$$\underset{n\to\infty}{Lim}\mathrm{E}\left[W_H^{CN-P}\right] - \underset{n\to\infty}{Lim}\mathrm{E}\left[W_H^{Ne}\right] = (1 - \lambda)\left(Q-\overline{C}-K\right) + \left(\overline{C}-\underline{C}\right);$$

and thank to Assumption 1 we know that this equation is always higher than 0.

iii) When P adopts either the Ne or the Au procedure, the score achieved by each firm is not a monotone function of its valuation. In fact, in the former case, firms are ordered with respect to their reliability, without any consideration for their production costs. By contrast, in the second case, firms are ranked according to their bids, that are function of their costs; therefore in this case the reliability factor plays no role. Consequently these two procedure are unable to select the highest valuation firm. This property characterises only the CN-P procedure; in this case in fact firms are ordered according to their valuation. *Q.E.D.*

Proof of Lemma 4: this lemma can be proved by means of the backward induction method: when A is corrupt firms know that the selected contractor in the negotiation phase may ask for a price equal to *Q,* being sure to be considered fully reliable by P. Consequently, the expected outcome of the negotiation stage is:

$$\mathrm{E}[\,p_C^{Ne}\,] = \lambda\overline{C} + (1-\lambda)Q$$

We can now define π_C^{Ne} as the expected profit of the firm that wins the bribery competition. It is simple to derive that:

$$\pi_C^{Ne}(C_i) = \lambda\overline{C} + (1-\lambda)Q - C_i$$

Obviously, the maximum bribe that a firm can offer to A cannot be higher than its expected profit in case of selection. Given the assumption of complete information with regard to firms' costs, we can deduce that the winner of the bribery competition is the highest expected profit firm, i.e. the lowest cost one. It will offer a price equal to the maximum willingness to pay for being contractor of its opponents, i.e. the expected profit of the second lowest cost firm; formally:

$$B(C_n) = \lambda\overline{C} + (1-\lambda)Q - C_{(n-1)}$$ *Q.E.D.*

Proof of Corollary 4: From lemma 4 we can derive that the expected bribe in case of negotiation is:

$$\mathrm{E}[\,B^{Ne}\,] = \lambda\overline{C} + (1-\lambda)Q - \mathrm{E}[\,C_{(n-1)}\,]$$

Given the assumption regarding the distribution of β, we can deduce that:

$$\phi^{Ne} = \text{Prob}\{\beta \le (1-\lambda)Q + \lambda\overline{C} - \text{E}[C_{(n-1)}]\} = \min\left[\frac{(1-\lambda)Q+\lambda\overline{C}-\text{E}\left[C_{(n-1)}\right]}{\overline{\beta}};1\right].$$

When A chooses to be corrupt the selection of the contractor is independent of the actual reliability of the firm. Consequently, the expected level of reliability of the selected firm is equal to the expected value of r. By substituting in equation [1] the expected values of both the price and the reliability factor we can then obtain P's expected welfare when the Ne procedure is adopted and A decides to be corrupt:

$$\text{E}[W_C^{Ne}] = \text{E}[Q - p - (1-r)K] = Q - \lambda\overline{C} - (1-\lambda)Q - K + \text{E}[r]K$$
$$= \lambda(Q - \overline{C} - K) - (1 - \lambda - \text{E}[r])K. \qquad \textit{Q.E.D.}$$

Proof of Lemma 5: The original proof can be found in the footnote 30 of Burguet et *al*. (2001). Here we report briefly their argument: if $\delta \ge K$, the lowest cost firm knows that no one of its opponents can outbid its offer, with or without the help of A. So it can capture the agent offering a bribe arbitrarily close to 0. Then it will win the auction by submitting a price equal to $C_{(n-1)} + K$. Conversely, if $K > \delta$, we obtain that the second lowest cost firm is willing to bribe no more than $K - \delta$; also in this case the lowest cost firm can out-bribe all its opponents by proposing a bribe arbitrarily higher than $K - \delta$, and then charging a price equal to $C_{(n-1)} + K$.

Proof of Corollary 5: From lemma 5 we can derive that the expected bribe in case of CN-P mechanism is:

$$\text{E}[B^{CN-P}] = \text{E}[(K-\delta)\text{Prob}\{\delta < K\}]$$

Given the assumption regarding the distribution of β, we can derive that:

$$\phi^{CN-P} = \text{Prob}\{\beta \le \text{E}[(K-\delta)\text{Prob}\{\delta < K\}]\} = \min\left[\frac{\text{E}\left[(K-\delta)\middle|\delta<K\right]\text{Prob}\{\delta<K\}}{\overline{\beta}};1\right].$$

Also in this case the selection of the contractor is independent of its actual level of reliability. Therefore the expected level of reliability is equal to the expected value of r. Moreover, the price offered by the winning firm is surely equal to $C_{(n-1)} + K$. By including these two results in equation [1] we obtain:

$$\text{E}[W_C^{CN-P}] = \text{E}[Q - p - (1-r)K] = Q - \text{E}[C_{(n-1)}] - 2K + \text{E}[r]K. \qquad \textit{Q.E.D.}$$

Proof of Corollary 6: From lemma 6 we know that in this equilibrium i) if $\delta \ge 3K$, then the winning firm offers a bribe arbitrarily close to 0; ii) if $\delta < 3K$, then both the firms randomise uniformly their bribe offer between 0 and $2K$. As explained in the appendix of Burguet et *al*. (2004), the probability of selection of a specific firm is independent of its bribe. Consequently, the expected bribe in this case is equal to:

$$\text{E}[B^{CN-C}] = \text{E}[K\,\text{Prob}\{\delta < 3K\}].$$

Given the assumption regarding the distribution of β, we can deduce that:

$$\phi^{CN-C} = \text{Prob}\{\beta \le \text{E}[K\,\text{Prob}\{\delta < 3K\}]\} = \min\left[\frac{\text{E}\left[K\middle|\delta<3K\right]\text{Prob}\{\delta<3K\}}{\overline{\beta}};1\right].$$

Also in this case the selection of the contractor does not depend on the actual reliability of the firms. Therefore, the expected level of reliability is equal to the expected value of r. As far as the expected price is concerned, we know by lemma 6 that if $\delta \ge 3K$, then the winning firm offers a price equal to $C_{(n-1)} - K$; conversely, if $\delta < 3K$, then both firms randomise their price offer according to the equations of lemma 6. We know that the probability of selection of a specific firm is independent of its actual offer. Consequently the expected price is equal to:

$$\text{E}[p_C^{CN-C}] = \text{E}[(C_{(n-1)} - K)\text{Prob}\{\delta \ge 3K\}] +$$
$$+\text{E}\left[\left\{\left(C_{(n)}+\frac{1}{3}\delta+B\left(C_{(n)}\right)+K\right)\left(\frac{1}{2}+\frac{\delta}{6K}\right)+\left(C_{(n-1)}-\frac{1}{3}\delta+B\left(C_{(n-1)}\right)+K\right)\left(\frac{1}{2}-\frac{\delta}{6K}\right)\right\}\text{Prob}\{\delta<3K\}\right]$$

Developing we obtain:

E[p_C^{CN-C}] = E[$(C_{(n-1)}$] – E[K Prob{$\delta \geq 3K$}] + E$\left[\left(2K+\frac{\delta}{2}-\frac{\delta^2}{18K}\right)\text{Prob}\{\delta<3K\}\right]$

By including the expected reliability and the expected price in equation [1] we achieve:

E[W_C^{CN-C}]= Q–E[$C_{(n-1)}$]–K+E[r]K+E[$K\,|\delta \geq 3K$]Prob{$\delta \geq 3K$}–E[$(2K+z)\,|\delta<3K$]Prob{$\delta<3K$}. *Q.E.D.*

Proof of Proposition 4: i) From the properties of order statistics we know that the higher the number of samples, the lower the expected value of $C_{(n-1)}$ and the differential δ. Hence, we can deduce that a stronger competition implies always a higher probability of corruption ii) by observing the equation of ϕ^{CN-P} it is straightforward to note that its value is a weakly increasing function of K. In fact, the higher K, the higher the probability that $K > \delta$ and the conditional expected value of the difference $K - \delta$. The same argument holds for ϕ^{CN-C} . iii) as far as ϕ^{Ne} is concerned, we can note that its value is an increasing function of the difference between the expected value of the $n\text{-}1^{th}$ order statistics and the upper bound of the distribution of C. From the properties of order statistics we know that this difference is increasing in the size of the support of the distribution. *Q.E.D.*

Proof of Proposition 5: i) the first part of proposition 5 can be proved by showing that the following condition holds:

E[K Prob{$\delta < 3K$}] $\geq$ E[$(K - \delta)$Prob{$\delta < K$}]

If $\delta \geq K$ then the r.h.s. of the inequality is equal to 0 and the inequality is surely true. If $\delta < K$ then we know that $K \geq K - \delta$ and Prob{$\delta < 3K$}$\geq$ Prob{$\delta < K$}, $\forall \delta, K$. The two sides are equal when $\delta = 0$.

ii) From the definition of ϕ^{CN-P} and ϕ^{Ne} we can note that the former probability goes to 0 as K goes to 0, while the latter one goes to 0 as Δ goes to 0. Moreover, from proposition 4, we know that ϕ^{CN-P} is a monotone increasing function of K, while ϕ^{Ne} is a monotone increasing function of Δ. Combining these properties we have proven the proposition. *Q.E.D.*

Proof of Proposition 6: i) By combining corollary 5 and 6 we have:

E[W_C^{CN-C}]– E[W_C^{CN-P}] = K + E[$K\,|\delta \geq 3K$]Prob{$\delta \geq 3K$}– E[$(2K+z)\,|\delta < 3K$]Prob{$\delta < 3K$}.

It is possible to note that this difference is surely negative if $3K > \Delta$. In fact, in that case Prob{$\delta \geq 3K$}= 0, Prob{$\delta < 3K$} = 1 and E[$(2K+z)\,|\delta < 3K$] = $2K$ + E[z]. Therefore, the higher the value of K as compared with the value of Δ, the more convenient the CN-P procedure as compared with the CN-C one. Conversely, if K goes to 0, then the difference goes to 0 too, but it is surely positive.

As far as the second part of the proposition is concerned, we can observe that:

E[W^{Au}]– E[W_C^{Ne}] = $(1-\lambda)Q + \lambda\overline{C}$ – E[$(C_{(n-1)}$];

E[W^{Au}]– E[W_C^{CN-P}] = K.

Both this difference are surely positive, whatever the value of K and Δ are. *Q.E.D.*