Dipartimento di Scienze Economiche
Università degli Studi di Firenze

Working Paper Series

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Working Paper N. 08/2013
June 2013

DISEI, Università degli Studi di Firenze
Via delle Pandette 9, 50127 Firenze, Italia
www.dse.unifi.it

Stampato in proprio in Firenze dal DISEI
(Via delle Pandette 9, 50127 Firenze) nel mese di Giugno 2013,
Esemplare Fuori Commercio Per il Deposito Legale
agli effetti della Legge 15 Aprile 2004, N.106

# Capabilities and human dilemmas: how to cope with incompleteness

Nicolò Bellanca and Mario Biggeri♦

Abstract

The main objective of this paper is to discuss the role of evaluative incompleteness in the work of Amartya Sen. In section 2, we consider Sen's distinction between optimising or perfect choices and maximising or acceptable choices, showing it is based on a restrictive interpretation of incompleteness. In section 3, we argue that this distinction does not take into account a third crucial category of human choices, the dilemmas, and we show they are based on the concept of incompleteness in a strict sense. Section 4 discusses the reason why in Sen's theoretical approach dilemmas and incompleteness appear to be somehow neglected. Then, in section 5 we advance an attempt to analyse human choices – both in the form of dilemmas or not – in conditions of incompleteness, while in section 6 we suggest a definition of freedom able to embrace also these dilemmatic choices. Finally, section 7 concludes.

JEL codes: A13; B59; D01; D63; I30; O10

Key words: evaluative incompleteness; Amartya K. Sen; dilemmas; freedom;

♦ Department of Economics, University of Florence. We thanks Matteo Aria, Pasquale De Muro and Andrea Ferrannini. Vittorioemanuele Ferrante has contributed to section 5; while Ian Carter contributed with comments and suggestions to section 6. We thanks the participants to the PRIN international workshop at Modena for their comments and suggestions (29-30 October 2012). We acknowledge the fund received by PRIN 2009 *Measuring human development and capabilities in Italy: methodological and empirical issues.* The usual disclaimer applies.

> The acceptability of evaluative incompleteness is indeed a central subject in social choice theory in general, and it is relevant to theories of justice as well
>
> Amartya Sen (2009: 105)

> It is important not to see the use of the capability approach as an "all or nothing" exercise. Indeed, the nature of interpersonal comparisons of well-being as well as the task of inequality evaluation as a discipline may admit incompleteness as a regular part of the respective exercises.
>
> Amartya Sen (1992: 48)

## 1. Introduction

The main objective of this paper is to discuss the role of evaluative incompleteness in the work of Amartya Sen. In section 2, we consider Sen's distinction between optimising or perfect choices and maximising or acceptable choices, showing it is based on a restrictive interpretation of incompleteness. In section 3, we argue that this distinction does not take into account a third crucial category of human choices – the dilemmas, and we show they are based on the concept of incompleteness in a strict sense. Section 4 discusses the reason why in Sen's theoretical approach dilemmas and incompleteness appear to be somehow neglected. Then, in section 5 we advance an attempt to analyse human choices – both in the form of dilemmas or not – in conditions of incompleteness, while in section 6 we suggest a definition of freedom able to embrace also these dilemmatic choices. Finally, section 7 concludes.

## 2. From perfect to acceptable choices

One of the analytical and interpretative pillars of Amartya Sen's theoretical approach can be identified in his distinction between perfect and acceptable choices. Indeed, Sen argues that only a complete and transitive ordering of a finite range of alternatives allows the identification of the optimal option – necessary for a perfect choice – through a process of one-to-one comparison between the alternatives. On the other side, if we consider a partial (i.e. transitive but incomplete) ordering, each or more alternatives are not dominated by any other, thus composing the maximal set– necessary for an acceptable choice (Sen, 2002, chapters 2 and 3). Therefore, the process of maximisation, limited to identify a range of alternatives not worse than any

other accessible option, differs from the process of optimisation, which aims at identifying an alternative at least as good as all the others, due to incompleteness.

The way the introduction of the idea of incompleteness is justified crucially shapes its relevance and implications. Sen[1] argues that the fundamental reason for incompleteness lies on the ambiguities and shades of human judgments on complex alternatives. The impossibility of completing an ordering does not impede, in a pragmatic way, to focus on its segments which can be clarified and shared, and thus become object of decision (Sen, 1992; 2009).

With this regard, the first step is the identification of the partial dominating ordering, where the agent considers *x* above *y* if and only if *x* is better than *y* on the basis of any desirable characteristic. In other words, the agent improves as he/she increases the amount of all his/her relevant alternatives, (starting from having goods, moving to opportunities/capabilities and functionings), independently from the relative weights he/she attaches to each option. Indeed, this partial order can be generalised event without elaborating a unique group of relative weights (Sen, 1970, chapters 7 and 7*; 1982, chapter 9).

Then, if the agent is internally animated by a plurality of evaluative criteria, « that the relative weitght to be attached to x vis-à-vis should be ½, 1/3, ¼ and 1/5, there is, then, an implicit agreement that the relative weight on x should not exceed V4, nor fall below M. But even this agreement will, in general, permit us to order pairs-—possibly many pairs—not covered by the dominance ranking. For example, with the weights specified, having one unit of x and two of y would be clearly better than having two units of x and one of y (even though neither pair dominates the other in the sense of having more of each x and y)» (Sen, 1992: 46-47). Therefore, if the agent selects the weights in a determined interval, he/she can reach a partial ordering of his/her alternatives, even without having identified the exact value of the weights in the interval. This process can substantially reduce the number of situations with no decision, though not completely eliminating them. Moreover, while the relatives weights and their relevant interval are selected by each individual according to his/her own reasoning, at an inter-agent and interactive level, a public discussion and democratic acceptance is required (Sen, 1999: 81).

A second step can be identified when the agent gives further specification to the possible weights, thus progressively reducing the interval. Here Sen (1970; 1982,

[1] Analysing Sen's point of view, we refer to evaluations and individual choices, even if the argument can be extended to embrace also social choices.

chapter 9) shows that usually the partial ordering reaches its completeness well before the system of weights becomes unique.

Sen's theoretical position can thus be synthesised in few words: «even with an incomplete ordering many decision problems can be adequately resolved, and even those that are not fully resolved can be substantially simplified {through the rejection of "dominated" alternatives)» (Sen, 1999: 313).

We can classify in 4 groups these "many problems" which can be resolved despite incompleteness. The first typology refers to a provisional incompleteness and it can be resolved implementing the information. «For example, having a medical intervention and enjoying a visit to a foreign country are two quite non-commensurate achievements, but a person may not have much problem in deciding which would be more valuable in her condition, and that judgement may of course vary with what she knows about her state of health and what her other concerns are» (Sen, 2009: 241)[2]. The second group derives from the *recognition* that some alternative are worse in a self-evident way. Therefore in this case, there is not a proper choice, but rather simply an conscious awareness. « The recognition of evident injustice in preventable deprivation, such as widespread hunger, unnecessary morbidity, premature mortality, grinding poverty, neglect of female children, subjugation of women, and phenomena of that kind does not have to await the derivation of some complete ordering over choices that involve finer differences and puny infelicities. Indeed, the overuse of the concept of justice reduces the force of the idea when applied to the terrible deprivations and inequities that characterize the world in which we live. Justice is like a cannon, and it need not be fired (as an old Bengali proverb puts it) to kill a mosquito» (Sen, 1999: 254). The third type of problems can be faced by selecting the best alternatives, though without putting effort in the search of the optimal alternative. « Indeed, it is not at all obvious why in making the judgement that some social arrangement X is better than an alternative arrangement Y, we have to invoke the identification that some quite different alternative, say Z, is the very 'best' (or absolutely 'right') social arrangement. In arguing for a Van Gogh over a Picasso we do not need to get steamed up about identifying the most perfect picture in the world, which would beat the Van Goghs and the Picassos and all other paintings in the world. » (Sen, 2009: 101). Finally, the fourth category requires an ex ante normative elaboration and/or a public deliberation, which define the important characteristics of

[2] Here Sen discusses about commensurability, and not about comparability. While the former requires a scale of unit values to measure the cardinal differences among the alternatives, the latter needs only an ordinal ranking. In this paper we focus on the comparability.

the alternatives, thus allowing their – at least partial – ordering. « The motivation underlying the approach of "development as freedom" is not so much to order all states-or all alternative scenarios-into one "complete ordering," but to draw attention to important aspects of the process of development, each of which deserves attention. Even after such attention is paid, there will no doubt remain differences in possible overall rankings, but their presence is not embarrassing to the purpose at hand » (Sen, 1999: 33).

In the following section, we will argue that this last typology of problems, which can be faced with Sen's method of partial ordering, neglects a crucial field where the incomparability shows up: the one of dilemmas.

## 3. Incompleteness and human dilemmas

The reasons behind incompleteness are, in our view, wider and deeper than those evoked by Sen. We thus suggest a perspective based on human dilemmas.

A dilemma derives from a conflict among objectives, each one achievable only if trade-offs or compromises with other objectives are refused. Antigone or Oedipus in the masterpiece of Sophocles, Hamlet or Lady Macbeth in the one of Shakespeare, are deeply involved in this kind of dilemma. Nonetheless, we focus here on a less famous example: "the donkey's dilemma", improperly attributed to the XIV century logician Jean Buridan.

Three different versions of this dilemma, with radically distinctive meanings, can be found in the literature. The first tells about a donkey which is indifferent between two sacks of oats and then it dies of hunger without taking a decision on the best option. In this case the agent aims at maximising its own pleasure but, facing very similar options, it has to refine its evaluation with such an effort that, in the meantime, it dies. The second version is the one presented by Sen, as the donkey had the opportunity to reach a better situation rather than dying of starvation. Indeed, any choice would have been a maximizing choice, and selecting a maximizing alternative, even if not the best, would have been quite meaningful (Sen, 2002). Finally, the third version tells about an exhausted donkey reaching a crossroads: on one side, it sees some oats, on the other there is water. At this time the donkey understands that, considering its extreme weakness, if it will eat oats, it would die of thirsty and vice versa. Its own rationality thus leads him to decide not to decide,

waiting at the crossroads the inevitable end of its life. The tragic point is then here that none option is the best, not because it does not equalized (it is indifferent) the other, but rather because *each one acquires value together with the other*, and choosing one automatically excludes the other. In this situation, as already in Sen's version, the alternatives cannot be ordered on the basis of a judgement criterion, i.e. the donkey is not able to define their order; nonetheless, differently from the interpretation advanced by Sen, the options here are incompatible. The donkey is then facing a dilemma.

Dilemmas are the "salt" of our existence, giving meaning to each one's biography. They are the choices valuing most for us, as they mark the turning point of our identitary path, concerning life and death, power and passions. They regard the choices characterising almost every masterpiece: Sophocles and Shakespeare, already mentioned, have simply expressed in a supreme way what everyone experiences. In more precise terms, *a choice becomes a dilemma when they agent presents different judgements, none dominating, none dominated, and thus justifying incompatible actions*. In a dilemma, deliberation is arbitrary, not following a judgement criterion, and it threatens the identitary profile of the agent: if the choices does not entail a clear meaning for him/her, who is he/her? It is indeed when facing a similar difficulty that usually the agent *decides to take a decision*: the predominance of one alternative on the other becomes the result of the deliberation itself; in other words, the deliberation becomes a way to give himself/herself sense , attempting to restore identitary unity and coherence. Carla Bagnoli (2006) advances the same argument regarding "symmetric dilemmas", i.e. the ones which, concerning equal amounts of the same value typology, entail commensurable alternatives and thus should be the easiest to be resolved. Let's imagine Emma and Martha – twins – with equal musical talent, but in a situation where their mother Ann can only afford to pay musical studies for one of them. In the simplest way, Beatrice could decide to "cut the knot", randomising the decision or delegating it to an external agent. Nonetheless, she matters about the way this deliberative deadlock is broken: to flip a coin, or to rely on some else's judgments, appear to her as iniquitous and non-sense procedures; Beatrice prefers a reason-based choice, giving her an "appropriate" resolution to the dilemma and establishing the identity Beatrice gives herself in the next period.

The theoretical point we are dealing with points out that, when facing dilemmatic situations, we challenge the meaning of our own actions and thus of our own identitary profile (Berlin, 2002: 222): in this situation we are confused about

ourselves because we do not have those evaluations necessary to act; to resolve the dilemma, the only way is to take a position and deliberate. However, an arbitrary choice is many times not enough, as we need to be recognised and to recognised ourselves while taking a decision and thus we have to select the reasons which assign preference to a certain path of choice compare to the others.

Our peculiarity, as human beings, consists indeed in being cultural animals, which cannot avoid to interpret the events (Weber, 1904; Macklem and Gardner, 2006). In name of this peculiarity, humans want to be rational (Elster, 2007), or, at least, reasonable, taking decisions according to reasons (Boudon, 1995). Reasons indeed allow interpreting the world, giving meaning to the events: an opaque, neutral, colourless and meaningless reality is not a reality where act is valuable. To put it briefly, choice, interpretation and action are interconnected. When dealing with genuine dilemmas, choice is not a mere neutral issue regarding calculus of relative benefits and costs, but rather it entails meta-decisions on the reasons of our acting. When dealing with human dilemmas, we move from a decision inspired by an evaluative or judgement criterion (based on utility, aesthetics, ethics, or others) to a decision based on reasons, *despite a judgement cannot be formulated*.

Briefly, dilemmas appear when: 1) reaching a value or an objective can only happen loosing something else and 2) the agent is not able to order the alternatives. The latter necessary characteristic (2) is eradicated in the incompleteness: the alternatives cannot be confronted and ordered, as none is preferable to the other, nor they are indifferent[3].

In order to fully understand dilemmas, we thus need to examine the incompleteness, discussing its conceptualisation, origin and relevance.

In the mainstream approach to rational choice, the agent faces a couple of alternatives: the set of his/her comparative judgements constitutes his/her preference relation (see Kreps, 1988; Gay, 1992). If the agent values *x* more than *y*, the first is strictly preferred to the second. If he/she prefers *x* to *y*, then to each other *z*, either *x* is preferred to *z*, or vice versa, or both judgments are valid. Due to this hypothesis of completeness in the preference relation, the agent is able to judge any alternatives included in the whole set of objects *X*, when evaluated two other alternatives to *X*:

[3] Sen (1987: 65) distinguishes between incompleteness (there is no binding reason for any of the two lines of action) and overcompleteness (there are reasons for each of the two lines of action, although conflicting). In our perspective, both incompleteness and hyper-completeness share the suspension of judgements between the alternatives; however, the incompleteness, adding the condition of incomparability between the lines of action, becomes a dilemma. Therefore, in Sen's perspective, we focus on hyper-completeness.

this one is either worse than the best one, or better than the worst one, or both. Therefore, the order relation configures a coherent and complete sequence, where we are able to compare any two elements within *X establishing which comes "first" and which "after"*. However, many social processes are animated by non-comparable alternatives, where the classic trichotomy – one is preferred to the other, or vice versa, or they are indifferent to each other – is not valid.

In these processes, incompleteness of the order relation prevails as, while we are in the condition to order a couple of *enough similar* alternatives, we are hesitant (we suspend the judgement) when facing a third alternative which is "too distant" or "too close" in qualitative terms.

Let's begin with an excessive unlikeness. We are, for instance, indifferent between buying a flat *x* in New York or a flat *y* in Boston. If we are offered a flat *z* in Boston, we keep being indifferent between *x* and *z*. Due to the transitivity in the indifference relation, we should also be indifferent between *y* and *z*. However, it could happen that a flat in Boston is preferred to another.

Let's have a look at an excessive likeness. When sequentially touching the burners 1, 2 and 3, I feel each one is cold. However, when touching first the burner 1 and then the number 3, I feel the difference as the latter is hotter. This entails that burner 2 is not comparable with the others. Indeed, when having a complete relation, as 1 proceeds 3, then either 1 should proceed 2, or vice versa, or both. But this is not the case.

These "common sense" cases point out that the possibility to order among all the alternatives applies until we believe to find adequate similarities among them. If the set X represents then our universe of social processes, we compare its elements *x*,*y* and *z* only if they seem to be either not excessively unlike, or not excessively similar between each other.

A decisive aspect entails that incompleteness does not derive from "rationality failures" to which it is possible to find a remedy: it does not origin from information gaps on alternatives, nor from the agent's limited ability to elaborate those information. Rather, it emerges when the agent assigns values based on different and contrasting points of view, which are difficult to be integrated in a coherent and complete judgment, the more the alternatives are for him/her, as we have seen, too similar or too unlike. This is a general property of qualitative judgments, which cannot always be translated in quantity comparisons: it is a *fundamental anthropological condition* of choice, which can be find in any human society. However, this is not

everything. There is at least another reason which leads incompleteness to be inevitable: «Typically, option preferences are induced by outcome preferences. I prefer one option because I prefer its outcome, that is, its expected utility, compared to that of other options. If the situation is one of uncertainty or ignorance rather than risk, however, I may not be able to compare the outcomes» (Elster, 2007: 208). In the case of uncertainty, «certain decisions that our individual is asked to make might involve highly hypothetical situations, which he will never face in real life; he might feel that he cannot reach an "honest" decision in such cases» (Aumann, 1962: 446). In the case of ignorance, when for instance we decide among many gastronomic menus, we usually consider a plurality of attributes: taste, quality, calorie, and so on. This circumstance can generate the non-comparability as discusses. To avoid it, let's imagine the agent considers only one criterion: "taste". Here, the non-comparability emerges anyway: indeed, «in order to choose, the agent must order the range of alternatives on the basis of his/her own preferences. However, in order to express a preference or indifference on the basis of "taste", he/she should have already "tasted" the different alternatives» (own translation from Laise, 1998: 43). In other words, he/she should have managed to overcome the ignorance characterising many new human experiences. Therefore, this second reason as well points out, evoking incompleteness and ignorance, that the former derives from the *anthropologic condition of choice*, and not simply from a formal characteristic, which can be introduced or removed to favour the analytical tractability of the economic model (for experimental tests of this argument, see Danan and Ziegelmeyer, 2004; Eliaz e Ok, 2006).

## 4. The fundamental conflicts between values and objectives: the human condition

Incompleteness is then a very diffused condition in human choices, as it tends to emerge when choices regard alternatives which are too similar, too unlike, uncertain or we are ignorant on. In addition, incompleteness is a necessary characteristics of human dilemmas, which tend to emerge when the agent protects his/her own integrity when facing choices regarding life, death, power and passions. In our view, it then derives that the link incompleteness-dilemmas is worthy to be at the centre of the theoretical debate. However, this is not Sen's position, which deals

with incompleteness "accommodating" it, erasing the human dilemmas and with them the incomparable alternatives.

To discuss it, let's consider a simple application of the capability approach to a dilemmatic situation. As renowned, in Amartya Sen's theory, the agent's wellbeing is defined both by a combination or vector of valued functionings (well-being achievements), and by the set of achievable functionings (well-being freedom). The latter – the set of possible combinations of effectively achievable functionings which can be selected by the agent – constitutes the set of capabilities or real freedoms. Let's suppose that the capability set includes only two vectors. One regards the set of available functionings for the agent if, living in Tibet,, he/she publicly accepts the Chinese government's rules. The other rather includes the set of available functionings to the agent when, living in Tiber, he/she openly profess him/herself Dalai Lama' cultist. The agent thus has to decide between a richer level of functionings – combined with the deletion of his/her own cultural-national identity, and a reduced level of functionings – where some are null in quantity but combined with of his/her own identitary integrity.

It's important to remark that in this situation functionings are not a list composed by independent processes between each other, nor by simply interdependent processes. Rather, each vector presents a hierarchical structure of lexicographic type: *first*, the agent take a position pro China or pro Dalai Lama; *then*, he/she gets access to one or other level of remaining functionings. For instance, who submit him/herself to the Chinese government gets access to working positions, education opportunities and health services, which he/she would be refused in the other case. The agent is thus aware – or he/she can easily experience – that his/her position yes-or-no, all-or-nothing, on a unique functioning determines every other dimension of his/her wellbeing. This is not a tragic choice on incompatible values/objectives; it is a dilemma, each time the two vectors appear non-comparable to the agent. In Isaiah Berlin's words (2002: 43), «namely that ends collide; that one cannot have everything. Whence it follows that the very concept of an ideal life, a life in which nothing of value need ever be lost or sacrificed, in which all rational (or virtuous, or otherwise legitimate) wishes must be capable of being truly satisfied - this classical vision is not merely Utopian, but incoherent. The need to choose, to sacrifice some ultimate values to others, turns out to be a permanent characteristic of the human predicament».

With regard to the couple of values/objectives composed by liberty and equality, Keith Dowding (2006: 332-335, added italics) argues: «One way of viewing Sen's development of the capability approach is as his attempt to bring together in one concept the demands for welfare and those of liberty. [...] The underlying idea of the capability approach is that we equalize across capability sets but allow liberty to flourish as individuals choose functionings from those sets. In that sense equality and liberty can be brought together under one approach. The problem from an egalitarian perspective, however, is that the choice of functionings affects the valuation of the relevant capability sets. [...] How successful I might be as a sportsman will be affected by who else chooses that life and how others value sports, as that valuation will determine my financial return and status in society. Choices affect the valuation of different capabilities; liberty affects planning for equality. [...] *Equality and freedom compete and that is about that*». Therefore, Sen manages to keep together liberty and equality (or other last values/objectives) – marginalising their possible incompatibility – only by denying the "human condition" mentioned by Berlin. Sen assumes that the choice of a vector of functionings does not modify the other vectors, within a *certain* capability set. Moreover, he assumes that any element in each vector is characterised by an independent level from the one of any other element, or it is interdipendent but not lexicographic superior. Both assumptions are not valid: the former because, as noted by Dowding, the choice of functionings generally influences the capability set; the latter because – this is our main argument – when facing a dilemma, many functionings of a vector depend on the choice of that unique functioning implicated in the dilemma.

## 5. Choice facing incompleteness

Having considered that non-comparability changes the nature of choices – dilemmatic or not, how can we consider it? Let's advance some initial arguments.

When preferences are complete, a problem of binding optimization is constituted by decisional variables (whose optimal value has to be determined), an objective function (indicating the functional relation between the decisional variables and certain variables whose value has to be maximised or minimised) and an acceptable set (which is the set of available alternatives to the agent). The *homo oeconomicus* maximises (the profits, the level of satisfaction deriving from

consumption, the social wellbeing) or minimises (the necessary costs to produce a certain amount of output) his/her objective function with certain constraints; from his/her point of view, a relative maximum is also an absolute maximum if the objective function is almost concave, and the acceptable set is convex. His/her behaviour is predictable: considering his/her preferences and the constraints' nature and assuming appropriate formal restrictions, his/her choice is unique.

However, this deterministic conception is not valid when facing incomparability. As the agents decides voluntarily, he/she avoids those alternatives which are dominated according to his/her own judgements. While, in the case of comparability, the non-dominated alternatives coincide with the dominant ones, indicating the best one among them (the maximum status), now the agent aspires only to lie in the set of maximal alternatives, which dominate all the others without any superiority among them.

In other words, while the agent in order to maximise adopts the rule to scale up the ordering of alternatives to reach *the highest point*, in the search of maximal options, the rule is to scale up *when* possible: his/her choices depends by the initial state and by the process to realize them. Nonetheless, this is not enough. The set of incomparable maximal alternatives entails a paradoxical nature: it is a choice set where the judgments are suspended. Indeed, here both the preference judgments (otherwise, they would not be maximal options) and the indifference judgements (an utilitarian judgment necessary to state "these two alternatives are equal" is not applicable in this hypothesis) are missing. The paradox is then in the circumstance that, when comparability is not applicable, the agent faces a choice among alternatives which he/she is not able to order and he/she is forced to take a decision without any criterion!

How can these reasonable choices without judgment happen? When the alternatives look incomparable to him/her, the agent can adopt four fundamental strategies. The first consists of *trying to introduce the comparability*. This can happen through arbitrary expedients, as criteria based on chance, caprice or custom. In the words of John Maynard Keynes (1973: 294), «Generally speaking, in making a decision we have before us a large number of alternatives, none of which is demonstrably more "rational" than the others, in the sense that we can arrange in order of merit the sum aggregate of the benefits obtainable from the complete consequences of each. To avoid being in the position of Buridan's ass, we fall back, therefore, and necessarily do so, on motives of another kind, which are not "rational"

in the sense of being concerned with the evaluation of consequences, but are decided by habit, instinct, preference, desire, will, etc.». This represents a weak theoretical answer, because the mentioned motives, though very relevant, do not appear to be object of a specific rigorous analysis: chance, caprice and custom appear indeed as exogenous factors, which justify an action without explaining it.

Alternatively, in the second strategy the comparability can be *restored through an objective logic of the structure*. Marx, for instance, «had warned that the transformational powers of money subverted reality, "confounding and compounding ... all natural and human qualities ... [money] serves to exchange every property for every other, even contradictory, property and object: it is the fraternization of impossibilities". As the ultimate objectifier – a "god among commodities" – money not only obliterated all subjective connections between objects and individuals, but also reduced personal relations to the "cash nexus" [...] As pure exchange value, money necessarily assumed an "unmeaning" form, which in turn neutralized all possible qualitative distinctions between commodities. [...] For Marx, money was thus an irresistible and "radical leveler", invading all areas of social life» (Zelizer, 1994: 7-8). Here it is not the agent to desire to evaluate heterogeneous alternatives with a monetary measure; rather, the incomparability of the choices is effectively reduced through a coercive institutional device, introduced by the capitalistic money. Another renowned example regards the elementary family relationships, which, according to Lévi-Strauss (1949), are positioned as an invariant structure, and the various particular-concrete systems of kinship are just its transformation in an algebraic sense. The reason organising this structure transcends not only the individual conscience and existence, but also the society and history, entailing a sort of collective unconscious (o *esprit*). Positions such as Marx's and Lévi-Strauss's do not deal with the problem of subjective responses to non-comparability of social alternatives, but rather they dissolve it invoking a deeper homogeneous reality.

The third strategy to restore the comparability lies in *showing how the power relations establish a criterion to which every alternative is compared*. A paradigmatic case is the theory of "governmentality" of Michel Foucault (2009). This neologism, deriving from the crasis between the term "government" and "rationality", indicates the diffusion of appropriate strategies and logics of dominium for the entire population. Foucault and his followers, such as Espeland and Stevens (2007) and Fourcade (2011), examine the practices through which politicians, ideologists, technicians, lawyers, professional economists, *et similia*, actively intervene to make

effective a way to compare and evaluate social goods. Their approach has the merit to express a conception of power not just punitive, coercive and violent, but rather able to make hegemonic the subjectivity. Nonetheless, they refuse the idea – which we want to push – that many relevant social processes can *treat the non-comparability without dissolving or eliminating it, through individuals' voluntary and aware choices*. This is the fourth strategy, the one less analysed in the literature which appears to us extremely relevant.

To explain in the simplest terms how an agent can take a decision according to reasons though judgments are missing i.e. when his/her alternatives are non-comparable maximal, it is necessary to distinguish two fundamental case: when the non-comparability of the maximal alternatives is total and when partial (the only one often treated by Sen). We are dealing here with the first case.

The access to a maximal option is not a steady-state, without an impulse to change and with block to further choices. Indeed, a maximal *z* according to the preference relation $P$ can lie on a path with further improvements, which includes an indifferent *x* to *z*, where options better than *x* exist but not than *z*. As in figure 1, the element *x* constitutes a "bridge" between a maximal according to $P$ and further improvements. The preference relation $P$ and its inverse (dispreference relation), jointly taken, constitute the judgment relation G. The residual of G indicates the set N of alternatives which cannot be judged. Being out of G, in N we can define an equivalence (or indifference) relation in terms of couple of alternatives judged by the agent with equal utility. It is instead possible to establish the indifference through a third alternative which can be accessed from them: therefore, in a space N without judgments, the indifference requisite does not mean anymore equal wellbeing or utility, but rather *having the same relation with other alternatives*. In figure 1, *x* and *z* have the same relation with the alternative *w*, or with the alternative *k*, and thus they are indifferent. Being non-comparable, *x* and *z* are not substitute between each other (according to a judgment); they are instead substitute as elements of the same indifference class. Therefore, the agent may desire to substitute the maximal *z* with the maximal *x*, as *x* allows moving in *y*, because a change (ignoring its utility) can be fostered from *x*. These are the choices which express the fourth strategy: the non-comparability of the alternatives cannot be eliminated, "judgments are suspended" with it and a choice criterion inspired by a rational utility calculus is missing. Nonetheless, the situation in which the agent is not able to improve (or to maintain the indifference), nor even to define the improvement (or indifference), does not

imply that he/she does not aspire to change: indeed, the change is a value itself for the opportunity to experiment and to widen the knowledge it leads to. Independently from whether it increases the direct or immediate agent's wellbeing (Hayek, 1960). The movement from *z* to *y*, through *x*, is the elementary mechanism to realise this strategy.

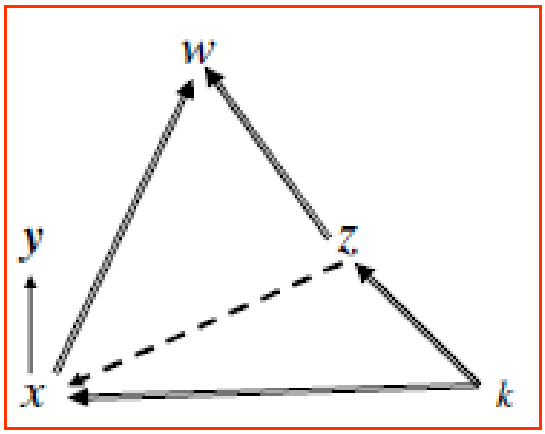


FIGURE 1

If $X$ is a set of elements of choice, the decision on them, faced by a single agent able to formulate value judgments, is traditionally based on a binary relation between the elements in $X$ called *preference* relation, which we can denote as $P$. Therefore: $P \subset X^2$ is the agent's binary relation of "strictly" asymmetric (thus non-reflexive) and transitive preference, interpreted in the usual way: $(y, x) \in P$, when the agent prefers *y* to *x*. $P^{-1} = \{(x, y) \mid (y, x) \in P\}$ is its inverse. $G = P \cup P^{-1}$ is the binary judgment relation, equally non-reflexive and transitive, but symmetric: $(y, x) \in G$ when the agent is able to express a judgement of "strict" preference between the two states. If $x \in X$, we define $P(x) = \{y \mid (y, x) \in P\}$the set of elements better than x according to P, and with $P^{-1}(x) = \{y \mid (y, x) \in P^{-1}\}$the set of elements worse than x. We then define $G(x) = \{y \mid (y, x) \in G\}$as the set of elements which can be judged, according to strict preference, with respect to *x*. Moreover, $N = X^2/G$ is the residual of the judgment; obviously, $(y, x) \in N$, when the agent does not judge the two elements according to preference. N is reflexive and symmetric, but not necessarily transitive. If N is transitive, it is an equivalence relations which identifies two classes in X, known as equivalence classes, which are not empty and separated, and their representative elements, and with them the whole classes, are linearly ordered according to P: P is then defined regular. Due to its completeness/bounded nature, P can thus represented numerically with a real function $u : X \rightarrow R$, which is

traditionally named utility function. Thanks to this, if (y, x) ∈ N, then u (y) = u (x), and this equality is interpreted as an "equal evaluation of wellbeing" of the elements by the agent, who judges them as indifferent.

Instead, if P is not regular, then it will always be possible to impose some transitivity constraint to the elements in N, in order to define a restriction which is an equivalence relation, eventually interpretable as indifference. Also in absence of a numeric representation, indeed, it would be reasonable to imagine the agent, though with non-regular preferences, able to, in certain cases, elaborate an indifference judgment between couples, respecting the transitivity requisite as it would be founded on a philosophy of equal evaluation. It seems reasonable, for instance, an indifference judgment, deriving naturally from the identity relation, in the case of a comparison of each element with itself. Generally, we can say that a set of couples in the residual N are characterised as indifferent if it respects the transitivity.

Various restriction criteria, sufficient to guarantee the transitivity, can be considered. Here we recall the equi-disposition criterion: two elements *x* and *y* are denoted as equi-disposed if and only if P (x) = P (y) and $P^{-1}$ (x) = $P^{-1}$ (y). In this case necessarily (y, x) ∈ N, as otherwise, for instance, (y, x) ∈ P and then y ∈ P (x), when obviously y /∈ P (y). If *x* and *y* are equi-disposed, and also *y* and *z*, then *x* and *z* are equi-disposed as well. Then (x, y) ∈ N and (y, z) ∈ N imply (x, z) ∈ N: the equi-disposition implies thus the transitivity. Particularly relevant is the superior equal-disposition, i.e. the equality of the best elements, which can be defined as $E^U$ = {(y, x) ∈ N | P (x) = P (y)}.

If Y ⊆ X, and P(x) ∩ Y = ∅, we can say that *x* is maximal (according to P) in Y, i.e. maximal tout court when Y = X. The existence of maximal elements is guaranteed by X's completeness/bounded nature. Then, we have that M is the set of maximal elements for the agent. If, for each x ∈ M, y ∈ q (x) is maximal, we can say that Q is equi-maximal, and we can define q (x) as the maximal equivalence class. If Q is equi-maximal, the it partition M: indeed, it cannot happen that any element from Q's partition includes both maximal and non-maximal. In the case of regular preferences, we have a unique maximal equivalence class; moreover, generally, if two elements are not in the same indifference class, then they are ordered according to strict preference and at least one is not maximal. If the preferences, instead, are not regular, then they cannot be ordered and they can both be maximal. It shows that the superior equi-disposition $E^U$ is the maximal equi-maximal equivalence, which

guarantees that the maximal elements belong to a unique equivalence class (for deeper insights, see Ferrante, 2012).

Let's resume. The loss of the residual's transitivity eliminates the possibility of a numeric representation of the preferences. This entails the important consequence of the impossibility to sustain a certain intrinsic concept of wellbeing, linked to each alternative status through the utility function, even in the case when it is possible despite the multiplicity of numeric representations with equal "utility" states. Indeed, when two elements are indifferent, they lead to the same wellbeing through any numeric representation. Rather, when we restore an indifference relation as transitive restriction on the residual, we cannot argue that two elements lead to the same wellbeing as they entail the same "utility", because we do not rely on any numeric representation. In the case of no-regular preferences then, when better options in a strict preference sense are not available and we are in a maximal situation, non-comparability with other elements does not allow further movements. Here it is valuable a definition of indifference whose interpretation is not the classic "equal wellbeing", but rather "equal relation" with other states. For instance, if $x \in q \in E$ and $y \in q \in E$, meaning that *x* and *y* belong to the same equivalence class, then they are indifferent not because they lead to the same wellbeing, but because each of them has access to the same best states, and they are the result of the same improvements. Alternatively, if $x \in q \in E^L$ and $y \in q \in E^L$, x and y are indifferent as they result from the same improvements, even if each of them has not access to the same best states. If we now introduce the possibility to substitute elements of the same indifference class among each other (even if we keep not allowing the substitution between non-comparable elements), i.e. if we construct the indifference relation with appropriate criteria to this substitutability hypothesis, and if, for instance, this construction leads us to consider a superior equi-disposed relation, we then have the possibility to substitute a maximal (according to P) with one of its non-maximal indifferent element, thus taking advantage of otherwise precluded improvements.

## 6. Freedom when facing dilemmas

Let's try to advance, considering the preceding insights, an "minimal" definition of freedom embracing also the dilemmatic situations. *An agent is free in sense [I] if*

*he/she can (1) decide which alternatives regard his/her choice in a non-dominated way by others; (2) decide among these alternatives. When facing dilemmas, an agent is free in sense [II] if he/she can (3) eliminate the dominated alternatives (according to his/her judgement); (4) decide among the non-comparable ones*. Sense [I] is representative of a theoretical position of "pure" negative freedom (Steiner, 1994; Carter, 1999); it is thus a "minimal" acceptation which could be substituted, without changing the remainder, with one more emphasising the "real freedoms" (together with Sen, see for example Pettit, 1997). Point (1), in its "minimal" terms, indicates the absence of *social* obstacles which *physically* impede to do something, and it excludes every reference to agent's believes, desires and values; point (2) indicates the mere possibility of choice/action. Rather, sense [II] is valid when the agent is facing dilemmatic situations. Within it, point (3) entails that those worsening alternatives according to the agent have to be removed: he/she would not select them voluntarily. The agent thus lies on the maximal states. The preference judgments function as a ladder which is thrown away as soon as we have reached the balcony. They indicate that the agents is "going up" according to his/her judgements, in order to remove the non-voluntary states and access to point (4): the possibility of choice in dilemmatic situations when judgments are missing.

In extreme synthesis, sense [I] tells: you are free if you have the possibility to decide/act; while sense [II] tells: if you are facing a dilemma, you are free if you have the possibility to decide/act even without judgments. These two parts of the definition of freedom are consistent one each other, but they do not entail different implications. We show them with four questions.

*1) If the agent has access to new non-maximal options, is he/she more free than before?*

He/she becomes more free, according to [I], as he/she expands the set of accessible alternatives, and thus of his/her possibility of actions. With respect to [II], it depends: if the set (2) expands reducing the sub-set (3), then he/she is less free; if instead the additive options to (2) are not taken from the maximal ones in (3), then his/her freedom in [II] does not change.

2) *If, passing from sub-set (3) to set (2), a greater number of alternatives become dominated, is the agent less free than before?*

He/she becomes more free [I], as he/she expands the set of accessible alternatives, and thus of his/her possibility of actions. He/she becomes less free [II], as the sub-set

of his/her non-comparable maximal alternatives is reduced, and thus his/her possibilities of action without judgments.

3) *If, passing from sub-set (3) to set (2), all the alternatives become dominated, does the agent become less free?*

He/she becomes more free [I], as he/she expands the set of accessible alternatives, and thus of his/her possibility of actions. Instead, he/she becomes not-free, as the sub-set of his/her non-comparable maximal alternatives disappears, and thus his/her possibilities of action without judgments.

4) *If someone orients the agent's preferences, widening the sub-set of his/her non-comparable maximal alternatives, does it make the agent more free?*

In respect to [I], it depends: if the sub-set (3) widens reducing set (2), then he/she is less free; if instead the additive option to (3) are not taken from alternatives in (2), then his/her freedom [I] does not change. He/she becomes more free [II], as the sub-set of his/her non-comparable maximal alternatives expands, and thus his/her possibilities of action without judgments.

This "complex" definition of freedom deepens the competition with the equality value/objective, on the line indicated by Berlin and Dowding, and it seems instead to take distance from the synthesis attempted by Sen with his capability approach. It is a perspective worthy to be explored with further research.

## 7. Conclusions

The topic of evaluative incompleteness, treated in terms of partial orderings, is a fundamental pillar of Amartya Sen's entire theoretical thinking, from the research on the individual rational choice, to that on social choice, to the capability approach, until the elaborations on justice. The critical perspective we have advanced regarding Sen's contribution differs from the more diffused and debated ones. As renowned indeed, the objections to Sen have principally regarded the partiality of the evaluative orderings he refers to, as well as the deriving weak evaluative metric.

The responses to these critiques – excluding the ones which have merely paraphrased Sen' s above-mentioned personal defence – have argued that the partial ordering needs somehow to be "completed" (for example, Nussbaum, 2000) and the metric needs to be refined taking into account the ambiguities and shades of the alternatives (for example, Chiappero Martinetti, 1994) and of the latent variables (for

example, Anand et al., 2011), or by "operationalising" through a wide battery of socio-economic indicators (for example, Burchardt e Vizard, 2011).

The answer we have advanced rather suggests that the ordering's incompleteness needs to be changed from partial to total; and within a really incomplete ordering, the reasons of choice have to be faced, not anymore relying on evaluations and judgements. To put it briefly, according to our interpretative line, Sen is stuck at the incompleteness's crossroads. This impedes him both to understand the relevance of dilemmatic conflicts between values/objectives, and to open to an analysis of the economic choices (§5) and of free actions (§6) able to fully embrace human dilemmas and the evaluative incompleteness characterising these choices.

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